Discrete Mathematics
Exam #1 (Chapters 1 & 2)
FALL 2012

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SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT!!!

1. For each of the following statements:
   a. Classify as universal, conditional, existential, or some such combination:
      1) All Computer Science upperclassmen at Suncoast are brilliant.

         Universal

      2) Every Computer Science senior has the ability to write programming code using Java.

         Universal Existential

      3) For all students in CS, if the CS student is planning on taking Discrete Math, then the CS student will have to comply with registration requirements mandated by FAU.

         Universal Conditional

      4) There is an 11^{th}-grade student in Discrete Math.

         Existential

      5) There is a final average grade for this course that must be greater than or equal to a grade of a "C".

         Existential Universal

      6) If a student is graduating in the Computer Science program, then the student is passing Discrete Math.

         Conditional $g \to p$

   b. For statement 6 above, identify the following statements as its negation, contrapositive, converse, or inverse.
      1) If a student is not passing Discrete Math, then the student is not graduating in the Computer Science program.

         $\sim p \to \sim g$ contrapositive

      2) A student is graduating in the Computer Science program and the student is not passing Discrete Math.

         $g \wedge \sim p$ negation

      3) If a student is passing Discrete Math, then the student is graduating in the Computer Science program.

         $p \to g$ converse

      4) If a student is not graduating in the Computer Science program, then the student is not passing Discrete Math.

         $\sim g \to p$ inverse

2. Let A = $\{x \in \mathbb{Z}^+ \mid x\ is\ a\ factor\ of\ 12\}$, B = $\{y \in \mathbb{Z}^+ \mid y\ is\ a\ factor\ of\ 15\}$, and C = $\{z \in \mathbb{R} \mid -8 \leq z \leq 13\}$. Define relations U, V, and W from A to B as follows:

$$For\ all\ (x, y) \in A \times B,$$

$(x, y) \in U$ means that $y - x > 0$

$(x, y) \in V$ means that $\frac{x}{y}$ is an integer

$(x, y) \in W$ means that $\frac{y}{x}$ is an integer

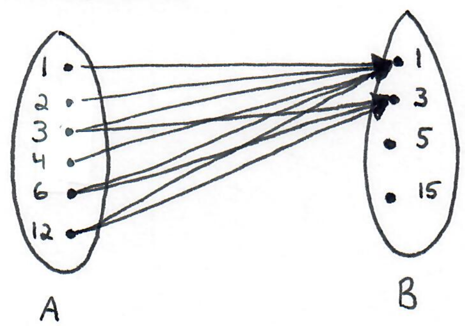
a. Is C $\subseteq$ A? Explain.

No. 1.1 $\in$ C, but 1.1 $\notin$ A.

b. Is A a proper subset of C? Explain.

Yes. Each of the elements of A = {1,2,3,4,6,12} appears in C, but C has more elements than those.

c. Draw an arrow diagram for V.



d. State the domain and co-domain of U.

Domain: {1, 2, 3, 4, 6, 12}

Co-Domain: {3, 5, 15}

e. Write W as a complete set of ordered pairs.

W = {(1,1), (1,3), (1,5), (1, 15), (3,3), (3, 15)}

3. Perform the indicated operations:

a. Write 100010010111_2 in decimal notation.

$1+2+4+0+16+0+0+128+0+0+0+2048 = \boxed{2199_{10}}$

b. Write 400_{10} in hexadecimal notation.

$400 - 1 \cdot 256 = 144 - 9 \cdot 16 = 0 - 0 \cdot 1 = 0$

1 9 0

$\boxed{190_{16}}$

c. Write $5D30C_{16}$ in decimal notation.

5	$5 \cdot 65536$	=	327 680
D	$13 \cdot 4096$	=	53 248
3	$3 \cdot 256$	=	768
0	$0 \cdot 16$	=	0
C	$12 \cdot 1$	=	12
			381708

$\boxed{381{,}708_{10}}$

d. Write $1A2_{16}$ in binary notation.

1	0001
A	1010
2	0010

$\boxed{110100010_2}$

4. Is the following statement a tautology? Justify your answer appropriately.

$$p \to (q \to r) \equiv (p \land q) \to r$$

p	q	r	$q \to r$	$p \to (q \to r)$	$p \land q$	$(p \land q) \to r$
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	T	T	F	T
T	F	F	T	T	F	T
F	T	T	T	T	F	T
F	T	F	F	T	F	T
F	F	T	T	T	F	T
F	F	F	T	T	F	T

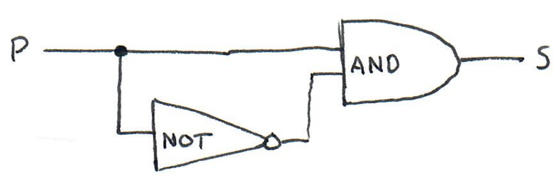
$\equiv$

t

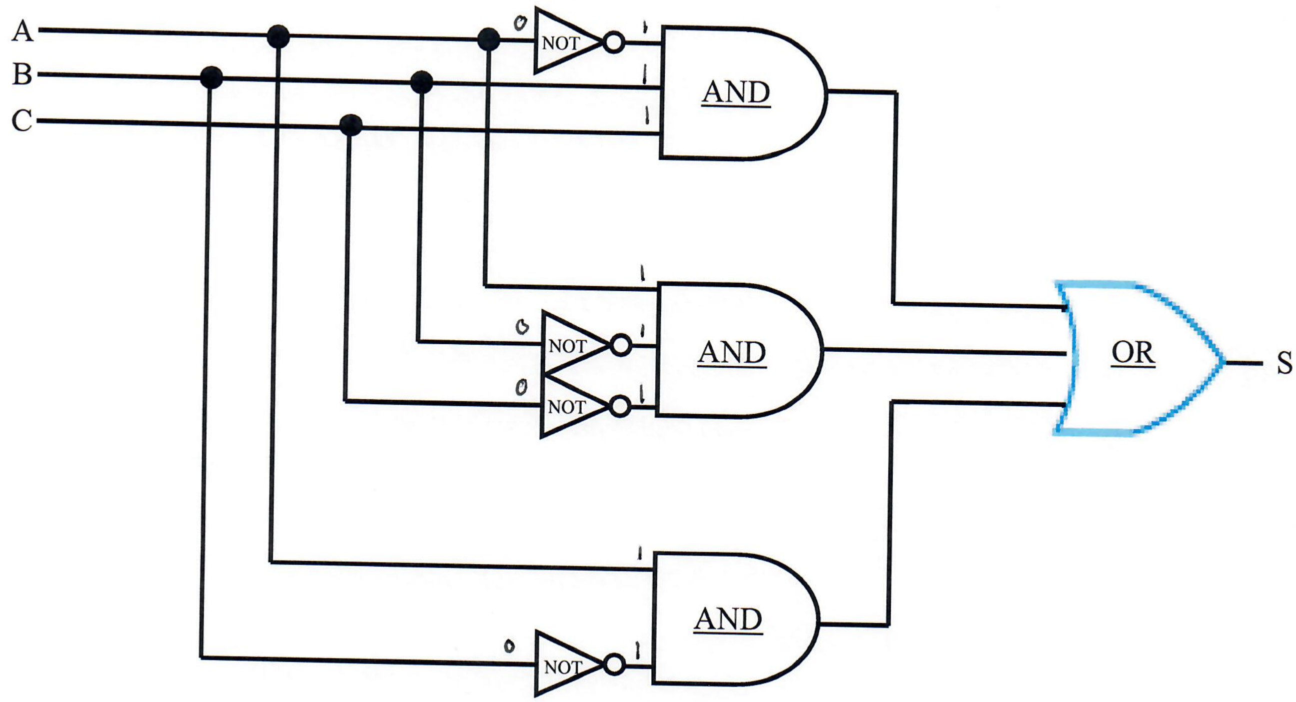
Because $p \to (q \to r)$ and $(p \land q) \to r$ are logically equivalent, the $\equiv$ operator will always return true. Thus, the statement is a tautology.

5. Draw a simple digital logic circuit that would result in a contradiction.

$p \land \sim p$ is a contradiction.



6. Given the following digital logic circuit:



a. Construct an input-output table representative of the circuit.

A	B	C	S
1	1	1	0
1	1	0	0
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

b. Write the Boolean expression that corresponds to the circuit.

$(A \wedge {\sim}B) \vee ({\sim}A \wedge (B \wedge C))$

7. Consider the following argument form:

(1) $s \to r$
(2) $(p \vee q) \to \sim r$
(3) $\sim s \to (\sim q \to r)$
(4) p
$\therefore q$

Use the valid argument forms to deduce the conclusion from the premises, giving a reason for each step. Assume all variables are statement variables.

p	(4)
$(p \vee q) \to \sim r$	(2)
$p \vee q$	(generalization)
$\sim r$	(modus poneus)
$s \to r$	(1)
$\sim s$	(modus tollens)
$\sim s \to (\sim q \to r)$	(3)
$\sim q \to r$	(modus ponens)
$\sim\sim q$	(modus tollens)
q	(double negative)

8. Determine whether the following argument is valid or invalid using a truth table. Assign statement variables, and include a few words with your truth table to explain why the argument is valid or invalid.

   Either the herbal remedy alleviated the symptoms, or the placebo effect alleviated the symptoms.

   If the placebo effect is responsible for easing the symptoms, then the herbal remedy is worthless.

   The herbal remedy alleviated the symptoms.

   Therefore, the herbal remedy is not worthless.

H: Herbal remedy alleviated

P: Placebo effect alleviated

W: Herbal remedy not worthless

$H \vee P$

$P \to \sim W$

H

$\therefore W$

H	P	W	$H \vee P$	$P \to \sim W$	H	W
T	T	T	T	F	—	—
T	T	F	T	T	T	F
T	F	T	T	T	T	T
T	F	F	T	T	T	F
F	T	T	T	F	—	—
F	T	F	T	T	F	—
F	F	T	F	—	—	—
F	F	F	F	—	—	—

True premises yield False conclusion.

The argument is invalid.

BONUS On the island of knights and knaves, every inhabitant is either a knight or a knave. Knights always tell the truth, while knaves always lie.

a. A stranger came to the island and encountered three inhabitants: Newcomer, Krumenacker, and Bobay. He asked Newcomer, "Are you a knight or a knave?" Newcomer mumbled an answer that the stranger could not understand. The stranger then asked Krumenacker, "What did she say?" Krumenacker replied, "Newcomer said that there is exactly one knight among us." Then Bobay burst out, "Don't believe Krumenacker, he is lying!" What can you conclude about these three inhabitants?

$B_T \rightarrow K_F \rightarrow$ Newcomer could have said anything besides "exactly one knight". We can not determine if it is true or false.

$\downarrow$ N_T $\quad\quad$ $\downarrow$ N_F

$B_F \rightarrow K_T \rightarrow$ Newcomer said there was one knight. If she's a knave, that statement would be true, so she is not a knave. If she's a knight, the statement would be false, so she is not a knight. Contradiction.

Bobay: Knight Krumenacker: Knave Newcomer: Unknown

b. The next day, the stranger encounters Bowers, who says "Either I am a knave or else two plus two equals five." What can you conclude about Bowers?

$2+2=5$ is a contradiction. If b represents Bowels being a knave:

$b \vee c$

b (identity)

His statement is equivalent to "I am a knave."

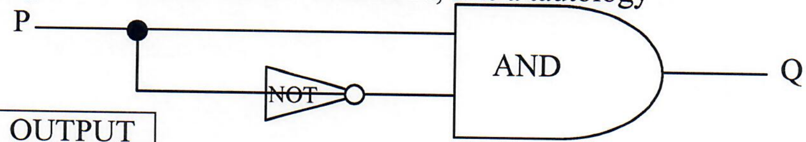
$B_T \rightarrow$ b is true $\rightarrow$ contradiction

$B_F \rightarrow$ b is false $\rightarrow$ contradiction

$\boxed{\text{No conclusion can be made.}}$

ANSWERS:

1. $a)$ $1 - universal,\ 2 - universal\ existential,\ 3 - universal\ conditional,\ 4 - existential$
   $5 - existential\ universal,\ 6 - conditional$
   $b)$ $1 - contrapositive,\quad 2 - negation,\quad 3 - converse,\quad 4 - inverse$
2. $a)$ No – rational numbers in C, only integers in A
   $b)$ Yes – $A \subseteq C;\ A \neq C;$ $d)$ Domain: $\{1, 2, 3, 4, 6, 12\};$ Co-domain: $\{3, 5, 15\}$
   $e)$ W = {(1,1), (1,3), (1,5), (1,15), (3,3), (3,15)}
3. $a)$ 2199_{10} $b)$ 190_{16} $c)$ $381{,}708_{10}$ $d)$ 000110100010_2
4. On your truth table, you will see these columns are identical. Since they are logically equivalent,
   $p \rightarrow (q \rightarrow r) \leftrightarrow (p \wedge q) \rightarrow r$ for all cases. Therefore, it is a tautology
5. Answers may vary. i.e.



6.

INPUT			OUTPUT
A	B	C	S
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

$b)$ $(\sim A \wedge B \wedge C) \vee (A \wedge \sim B \wedge \sim C) \vee (A \wedge \sim B \wedge C)$ $\quad OR \quad$ $(\sim A \wedge B \wedge C) \vee (A \wedge \sim B)$

7. $p\ (4);\ \therefore p \vee q\ (generalization)$
   $(p \vee q) \rightarrow \sim r\ (2);\ p \vee q\ (previous);\ \therefore \sim r\ (modus\ ponens)$
   $s \rightarrow r\ (1);\ \sim r\ (previous);\ \therefore \sim s\ (modus\ tollens)$
   $\sim s \rightarrow (\sim q \rightarrow r)(3);\ \sim s\ (previous);\ \therefore \sim q \rightarrow r\ (modus\ ponens)$
   $\sim q \rightarrow r\ (previous);\ \sim r\ (previous);\ \therefore \sim(\sim q)(modus\ tollens)\ \therefore q\ (negation)$
8. Label the following:
   HR – herbal remedy works; PE – placebo works; HW – herble remedy worthless
   Convert the statements to the following:

   $HR \vee PE$
   $PE \rightarrow HW$
   HR
   $\therefore \sim HW$

(INVALID ARGUMENT – true premises yield false conclusion)

POSSIBLE COMBINATIONS			PREMISES			CONCLUSION
HR	***PE***	***HW***	$HR \vee PE$	$PE \rightarrow HW$	***HR***	$\sim HW$
F	F	F	F			
F	F	T	F			
F	T	F	T	F		
F	T	T	T	T	F	T
T	F	F	T	T	T	F
T	F	T	T	T	T	
T	T	F	T	F	T	F
T	T	T	T	T	T	