Discrete Mathematics
Exam #1 (Chapters 1 & 2)
FALL 2013

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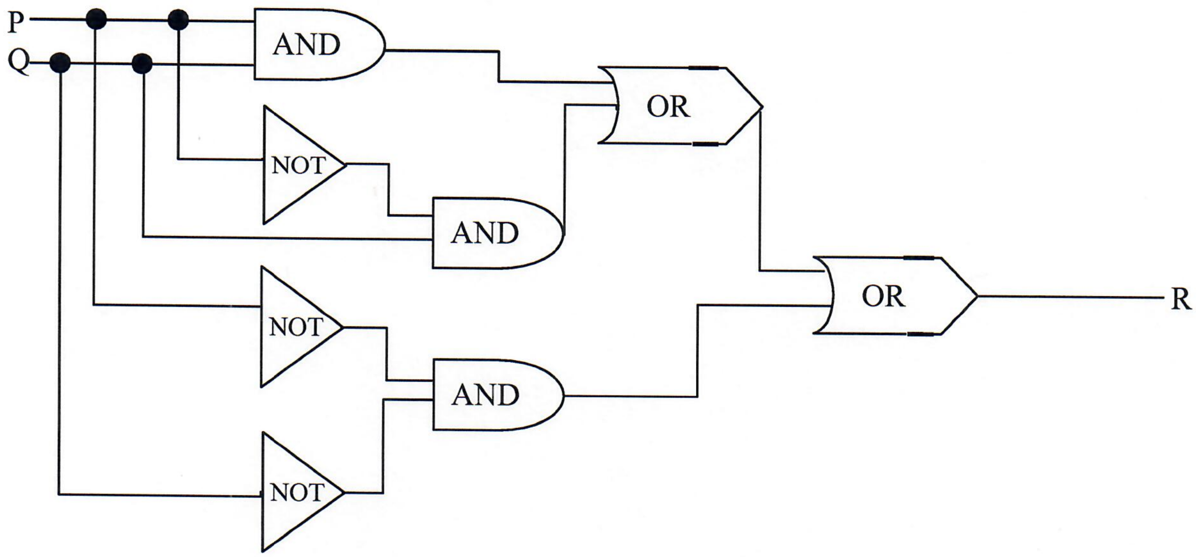
Date: 2017-02-04

Period: 7

SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT!!!

1. For the following digital logic circuit:



a. Construct an input-output table representative of the circuit.

P	Q	R
1	1	1
1	0	0
0	1	1
0	0	1

b. Write the Boolean expression that corresponds to the circuit.

$\sim P \vee (P \wedge Q)$

c. Simplify the Boolean expression for this circuit.

$(\sim P \vee P) \wedge (\sim P \vee Q)$ distribute

$t \wedge (\sim P \vee Q)$ identity

$\sim P \vee Q$

2. Let A = $\{x \in \mathbb{Z}^+ \mid -1 \le x \le 5\}$, B = $\{y \in \mathbb{Z}^+ \mid y \textit{ is a factor of } 30\}$, and C = $\{z \in \mathbb{Q} \mid -1 \le z \le 7\}$. Define relations U, V, and W from A to B as follows:

$$\textit{For all } (x,y) \in A \times B,$$
$$(x,y) \in U \text{ means that } y - 10x > 0$$
$$(x,y) \in V \text{ means that } \frac{x}{y} \text{ is an integer}$$
$$(x,y) \in W \text{ means that } \frac{y}{x} \text{ is an integer}$$

a. Is C ⊆ A? Explain.

No. $\frac{3}{2} \in C$ and $\frac{3}{2} \notin A$.

b. Is A a proper subset of C? Explain.

Yes. $\forall x \in A$, $x \in C$. Also, C contains additional values (i.e. $3/2$). Thus A is a proper subset of C.

c. True or False: $\{4, 2\} \in V$. Explain.

True. 1) $4 \in A$
2) $2 \in B$
3) $\frac{4}{2}$ is an integer.

All criteria met.

d. State the domain and co-domain of U.

Domain: $\{1, 2\}$

Co-Domain: $\{1, 2, 3, 5, 6, 10, 15, 30\}$

e. Write W as a complete set of ordered pairs.

W = {(1,1), (1,2), (1,3), (1,5), (1,6), (1,10), (1,15), (1,30),
(2,2), (2,6), (2,10), (2,30), (3,3), (3,6), (3,10),
(3,15), (3,30), (5,5), (5,15), (5,30)}

4. Classify the following statements as a contradiction, a tautology, or neither. Justify your answer appropriately.

a. $p \to (q \to (p \wedge q))$

$p \to (\sim q \vee (p \wedge q))$	$p \to q \equiv \sim p \vee q$
$p \to ((\sim q \vee p) \wedge (\sim q \vee q))$	distribute
$p \to ((\sim q \vee p) \wedge t)$	negation
$p \to (\sim q \vee p)$	identity
$\sim p \vee (\sim q \vee p)$	$p \to q \equiv \sim p \vee q$
$(\sim p \vee p) \vee \sim q$	associative
$t \vee \sim q$	negation
$\boxed{\text{tautology.}}$	identity

b. $(p \vee q) \to (q \to (p \wedge q))$

p	q	$p \vee q$	$p \wedge q$	$q \to (p \wedge q)$	$(p \vee q) \to (q \to (p \wedge q))$
T	T	T	T	T	T
T	F	T	F	T	T
F	T	T	F	F	F
F	F	F	F	T	T

$\boxed{\text{The statement is neither a tautology nor a contradiction.}}$

c. $p \to (q \to \sim(p \vee q))$

p	q	$\sim(p \vee q)$	$q \to \sim(p \vee q)$	$p \to (q \to \sim(p \vee q))$
T	T	F	F	F
T	F	F	T	T
F	T	F	F	T
F	F	T	T	T

$\boxed{\text{Neither.}}$

5. Classify each of the following statements as universal, conditional, existential, or some such combination, and determine whether the statements are true or false.

   a. All half-adders are constructed by four logic gates.

      Classification Universal

      (circle one) TRUE FALSE

   b. For all three-digit binary numbers, a parallel adder is composed of three full-adders (one for each pair of digits).

      Classification Universal existential

      (circle one) TRUE FALSE

   c. For all binary numbers, if constructing a circuit to add a two-digit binary number to a three-digit binary number, a full-adder is sufficient..

      Classification universal conditional

      (circle one) TRUE FALSE

6. Consider the following argument form:

(1) $u \to r$

(2) $(r \wedge s) \to (p \vee t)$

(3) $q \to (u \wedge s)$

(4) $\sim t$

(5) q

$\therefore p$

Use the valid argument forms to deduce the conclusion from the premises, giving a reason for each step. Assume all variables are statement variables.

$q \to (u \wedge s)$	(3)
q	(5)
$\therefore u \wedge s$	modus ponens
$u \to r$	(1)
u	Specialization
$\therefore r$	modus ponens
$(r \wedge s) \to (p \vee t)$	(2)
s	Specialization
$r \wedge s$	Conjunction
$\therefore p \vee t$	modus ponens
$\sim t$	(4)
$\therefore p$	elimination

7. Determine whether the following argument is valid or invalid using a truth table. Assign statement variables, and include a few words with your truth table to explain why the argument is valid or invalid. (Identify in your truth table which Boolean expressions are referencing premises and which is the conclusion.)

Either Lucic stole his phone or Todor stole his phone.
If Lucic looks guilty, then he stole his phone.
Lucic doesn't look guilty.
Therefore, Todor stole his phone.

L: Lucic stole his phone
T: Todor stole his phone
G: Lucic looks guilty

$L \vee T$
$G \to L$
$\sim G$
$\therefore T$

				premises		conclusion
L	T	G	$L \vee T$	$G \to L$	$\sim G$	T
T	T	T	T	T	F	
T	T	F	T	T	T	T
T	F	T	T	F		
T	F	F	T	T	T	F
F	T	T	T	T	F	
F	T	F	T	T	T	T
F	F	T	F			
F	F	F	F			

The truth table has true premises and a false conclusion @ line 4.

$\therefore$ $\boxed{\text{The argument is invalid.}}$

BONUS: On the island of knights and knaves, every inhabitant is either a knight or a knave. Knights always tell the truth, while knaves always lie.

A stranger came to the island and encountered five inhabitants: Gina, Vinny, Fredo, Louie and Bianca. Gina says that Bianca could say that Vinny is a knight. Vinny claims, "Louie and I are not the same." Fredo says that Louie is a knave. Louie says "At least one of the following is true: that I am a knight or that Bianca is a knave." Bianca says that Gina is a knave or Fredo is a knave.

Determine who in this group is a knave and who is a knight. Defend your answer.

Fredo_T

$\downarrow$

Louie_F

$\downarrow$

Louie not Knight & Bianca Knight

$\downarrow$

$B_T \dashrightarrow \sim G \vee \sim F$

F

$\therefore \sim G$

$\downarrow$

G_F

$\downarrow$

Gina is lying, so she can not say Vinny is a knight. This means Vinny is a Knave.

$\downarrow$

V_F

$\downarrow$

Him and Louie are the same, so he's lying, which checks out.

Fredo: Knight
Bianca: Knight

Louie: Knave
Gina: Knave
Vinny: Knave

ANSWERS:

1. a)

INPUT		OUTPUT
P	Q	R
0	0	1
0	1	1
1	0	0
1	1	1

   b) $(P \wedge Q) \vee (\sim P \wedge Q) \vee (\sim P \wedge \sim Q)$; c) $\sim P \vee Q$

2. a) No – $6 \in C, 6 \notin A$; b) Yes – $A \subseteq C, A \neq C$; c) False – don't confuse $\{4,2\}$ with $(4,2)$
   d) Domain $\{1,2\}$, Co-domain $\{15,30\}$;
   e) $\{(1,1),(1,2),(1,3),(1,5),(1,6),(1,10),(1,15),(1,30),(2,2),(2,6),(2,10),(2,30),(3,3),(3,6),(3,15),(3,30),(5,5),(5,10),(5,15),(5,30)\}$
3. a) 101001011010; b) $B35_{16}$; c) 64_{16}
4. a) TTTT – tautology (true for all cases); b) TFTT – neither; c) TTTF – neither
5. a) universal, TRUE; b) universal existential, FALSE; c) universal conditional, FALSE

6.

q	(E)
$q \to u \wedge s$	(C)
$\therefore u \wedge s$	(modus ponens)
$u \wedge s$	(previous)
$\therefore u$	(specialization)
$u \to r$	(A)
u	(previous)
$\therefore r$	(modus ponens)
$u \wedge s$	(previous)
$\therefore s$	(specialization)
r	(previous)
s	(previous)
$\therefore r \wedge s$	(conjunction)
$r \wedge s$	(previous)
$(r \wedge s) \to (p \vee t)$	(B)
$\therefore p \vee t$	(modus ponens)
$p \vee t$	(previous)
$\sim t$	(E)
$\therefore p$	(elimination)

7. Label the following:
   LS – Lucic stole phone; TS – Todor stole phone; LG – Lucic looks guilty
   Convert the statements to the following:

   $LS \vee TS$
   $LG \rightarrow LS$
   $\sim LG$
   $\therefore \sim TS$ (INVALID ARGUMENT – true premises yield false conclusion)

POSSIBLE COMBINATIONS			PREMISES			CONCLUSION
LS	TS	LG	$LS \vee TS$	$LG \rightarrow LS$	$\sim LG$	$\sim TS$
F	F	F	F			
F	F	T	F			
F	T	F	T	T	T	T
F	T	T	T	F		
T	F	F	T	T	T	F
T	F	T	T	T	F	
T	T	F	T	T	T	T
T	T	T	T	T	F	

8. Bianca & Fredo – knight; Gina, Vinny, Louie – knave