Discrete Mathematics
Exam #2 (Chapters 3 & 4)
FALL 2012

Name: Jacob Magnuson
Date: 2017-02-04
Period: 7

SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT!!! <u>**NO CALCULATORS!**</u>

1. Use the Euclidean algorithm to find the following:
   a. GCD(1739, 29341)

16
1739 | 29341
− 1739
11951
− 10434
1517

→ 1
1517 | 1739
− 1517
222

→ 6
222 | 1517
− 1332
185

→ 1
185 | 222
− 185
37

→ 5
37 | 185
− 185
0 ✓

$\boxed{37}$

b. GCD(431, 29)

14
29 | 431
− 29
141
117
24

→ 1
24 | 29
− 24
5

→ 4
5 | 24
− 20
4

→ 1
4 | 5
− 4
1

→ 4
1 | 4
− 4
0 ✓

$\boxed{1}$

2. Suppose $m \in \mathbb{Z}$. If $m \bmod 8 = 7$, what is $5m \bmod 8$?

$$m \bmod 8 = 7$$
$$5m \bmod 8 = 7 \cdot 5 \bmod 8$$
$$= 35 \bmod 8$$
$$= \boxed{3}$$

3. Today is Tuesday, October 30, 2012, and this year was a leap year. While you don't know where you will be on October 30, 2022, what day of the week will it be? Justify your answer using the quotient-remainder theorem.

3650 days + 2 leap days = 3652 days

$$3652 \bmod 7 \xrightarrow{-7\cdot 500} 152 \bmod 7 \xrightarrow{-7\cdot 20} 12 \bmod 7 \xrightarrow{-7} 5$$

$3652 \bmod 7 = 152 \bmod 7 = 12 \bmod 7 = 5$

Tuesday + 5 days

Tu W Th F Sa Su

$\boxed{\text{Sunday.}}$

4. For the following statement: $\forall$ *real number* x, *if* $-1 < x < 0$ *then* $x + 1 > 0$. Write the negation of this statement.

$$\exists x \in \mathbb{R} \mid -1 < x < 0 \text{ and } x + 1 \leq 0$$

5. Use the definitions and properties of even and odd integers to determine whether the following statements are true or false:

a. If m and n are integers, $6m^2 + 34n - 18$ is an even integer.

$6m^2 + 34n - 18 = 2(3m^2 + 17n - 9)$ Factoring

Let $k = 3m^2 + 17n - 9$.

$k \in \mathbb{Z}$ properties of integers

$6m^2 + 34n - 18 = 2k$ substitution

$2k$ is even Def. of even

$\therefore 6m^2 + 34n - 18$ is even. Transitivity.

b. For all integers a, b, and c, if $a|b$ and $a|c$ then $a|(2b - 3c)$.

$(a|b) \rightarrow b = ax,\ x \in \mathbb{Z}$ Def. of divisibility.

$(a|c) \rightarrow c = ay,\ y \in \mathbb{Z}$ "

$2b - 3c = 2(ax) - 3(ay)$ Substitution

$= a(2x - 3y)$ Factoring

$a \mid (a(2x - 3y))$

$\therefore a \mid (2b - 3c)$

c. The sum of any three consecutive even integers is divisible by 6.

Let $a \in \mathbb{Z}$.

$2a$ is even. Def of even.

Adding 2 will produce the next even integer.

$2a + (2a+2) + (2a+2+2) = 6a + 6$

$= 6(a+1)$

$6 \mid (6(a+1))$

$\therefore$ 6 divides any three consecutive even integers.

6. Determine whether the statement is true or false, justifying your answer with a proof or counterexample as appropriate:

a. Every positive integer can be expressed as a sum of three or fewer perfect squares.

$1 = 1^2$

$2 = 1^2 + 1^2$

$3 = 1^2 + 1^2 + 1^2$

$4 = 2^2$

$5 = 2^2 + 1^2$

$6 = 2^2 + 1^2 + 1^2$

$7 = \underbrace{2^2 + 1^2 + 1^2 + 1^2}$

False.

The theorem is false. 7 can not be written as the sum of three or fewer perfect squares.

b. For all integers n, $n^2 - n + 11$ is a prime number.

Let $n = 11$.

$(11)^2 - 11 + 11 = 11^2 = 121$

$11 \mid 121$

121 is composite

$\therefore$ The theorem is False.

7. Is the following argument valid or invalid? Justify your answer.

   All real numbers have nonnegative squares.
   The number i has a negative square.
   Therefore, the number i is not a real number.

R: a number is real
S: a number has a nonnegative square

(1) $R \to S$
(2) $\sim S$
$\therefore \sim R$ modus tollens

$\boxed{\text{Valid.}}$

8. Reorder the premises and state the conclusion that follows logically from the argument. (It may be helpful to rewrite some of the statements in if-then form and replace some statements with their contrapositives.

   1. Animals, that do not kick, are always unexcitable
   2. Donkeys have no horns
   3. A buffalo can always toss one over a gate
   4. No animals that kick are easy to domesticate.
   5. No hornless animal can toss one over a gate
   6. All animals are excitable, except buffalo

(2) Donkeys have no horns.
(5) No hornless animal can toss one over a gate.
(3) A buffalo can always toss one over a gate.
Note: This means a buffalo has horns → buffalos ≠ donkeys.
(6) All animals are excitable, except buffalo.
Note: This means donkeys are excitable.
(1) Animals that don't kick are always unexcitable.
Note: $\sim K \to \sim E$ contrapositive $E \to K$
therefore donkeys kick.
(4) No animals that kick are easy to domesticate.

$\boxed{\therefore \text{ Donkeys are not easy to domesticate.}}$

9. A Suncoast student has to make some decisions when entering this school: their magnet program (CS, MSE, IB, or IIT), their foreign language (Spanish or French), an elective credit (fine arts or business/vocational), and how they will be satisfying their Physical education requirement (taking the class here, taking it online through FLVS, or participating in a varsity sport). Four students enter Suncoast and make the following selections:

Learn to count, Oddi...

- Kayla: IB, French, chorus, varsity volleyball
- Ming: MSE, French, PE on FLVS
- Herbert: CS, Spanish, computer programming, PE on FLVS
- Alex: IIT, Spanish, drafting, PE with Ms. McCann at Suncoast
- Adit: MSE/IB, Spanish, French, programming, PE on FLVS

Write each of the following statements informally and find its truth value. If no truth value exists, answer accordingly.

a. ∃ a decision *D* such that ∀ students *S*, *S* chose *D*.

Every student chose the same option for one of the four decisions.

This is false.

b. ∃ a student *S* such that ∀ decisions *D*, *S* chose *D*.

There is a student that chose every single thing there is to choose.

This is false.

c. ∃ a student *S* such that ∀ categories *C*, ∃ a decision *D* in *C* such that *S* chose *D*.

There is a student that made a decision in each category.

This is true. Kayla made a decision for each category.

d. ∀ students *S* and ∀ categories *C*, ∃ a decision *D* in *C* such that *S* chose *D*.

Every option from every category was chosen by at least one student.

This is false. Adit chose MSE/IB, which is not a decision from any category.

ANSWERS:

1. $a.\ 37 \quad b.\ 1$
2. 3
3. Sunday
4. $\exists\, a\ real\ number\ x\ such\ that -1 < x < 0\ and\ x + 1 \leq 0$
5. $a.\ 6m^2 + 34n - 18;\ 2(3m^2 + 17n - 9)\ (factoring);\ k = 3m^2 + 17n - 9, k \in \mathbb{Z}\ (properties\ of\ integers);\ 6m^2 + 34n - 18 = 2k\ (substitution);\ 2k\ is\ even\ (definition\ of\ even);\ therefore\ 6m^2 + 34n - 18\ is\ even\ integer\ (transitivity)$

   $b.\ (a|b) \to b = ax, \quad x \in \mathbb{Z}\ (definition\ of\ divisibility);\ (a|c) \to c = ay, y \in \mathbb{Z}\ (definition\ of\ divisibility);\ 2b = 2ax\ (substitution);\ 3c = 3ay\ (substitution);\ 2b - 3c = 2ax - 3ay\ (substitution);\ (a|a(2x - 3y));\ (a|(2b - 3c))$

   $c.\ let\ a, b, c\ \in \mathbb{Z}\ (definition\ of\ divisibility);\ a = 2k, b = 2k + 2, c = 2k + 4, k \in \mathbb{Z}\ (definition\ of\ even\ and\ consecutive;\ a + b + c = 6q + 0, q \in \mathbb{Z};\ 2k + 2k + 2 + 2k + 4 = 6q + 0;\ 6k + 6 = 6q;\ 6(k + 1) = 6q;\ 6k + 6\ is\ divisible\ by\ 6;\ therefore\ a + b + c\ is\ divisible\ by\ 6$
6. $a.\ False\ (counterexample, x = 7);\quad b.\ False\ (counterexample,\ n = 11)$
7. Valid – modus tollens
8. 2, 5, 3, 6, 1, 4 – Donkeys are not easy to domesticate
9. $a.\ False;\ b.\ False;\ c.\ True;\ d.\ False$