Discrete Mathematics
Exam #2 (Chapters 3 & 4)
FALL 2013

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SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT!!! **NO CALCULATORS!**

1. Use the Euclidean algorithm to find the greatest common divisor of 8,529 and 17,432.

$17432 \div 8529 = 2$: $17432 - 17058 = 374$

$8529 \div 374 = 22$: $8529 - 748 = 1049$, $1049 - 748 = 301$

$374 \div 301 = 1$: $374 - 301 = 73$

$301 \div 73 = 4$: $301 - 292 = 9$

$73 \div 9 = 8$: $73 - 72 = 1$

$9 \div 1 = 9$: $9 - 9 = 0$

$\boxed{0}$

2. Today is Thursday, October 17, 2013. Given that 2012 was the last leap year we had, what day of the week was July 8, 1978? Justify your answer using the quotient-remainder theorem. (Hint: Think about this works in reverse, and how it would effect your answer if we would have started on July 8th to predict today's day of the week.)

7/8/1978 → 7/8/2013 → 10/17/2013

$35 \cdot 365$
$+ 9$ LD
$\equiv 2 \bmod 7$

Aug: 31
Sep: 30
$+17$
$+23$
$101 \equiv 3 \bmod 7$

Count back $2+3=5$ days.

Sa Su M Tu W Th

$\boxed{\text{Saturday}}$

3. Use the definition of mod to determine the following:

a. Suppose $p \in \mathbb{Z}$. If $4p\ mod\ 12 = 8$, what is $p\ mod\ 3$?

$$\frac{4p}{4} \text{ mod } \frac{12}{4} = \frac{8}{4}$$

$$p \text{ mod } 3 = \boxed{2}$$

b. Suppose $p \in \mathbb{Z}$. If $p\ mod\ 10 = 8$, what is $p\ mod\ 5$?

$$p \text{ mod } 10 = 8$$

$$p = 10k + 8,\ k \in \mathbb{Z}$$

$$p = 5(2k) + 5 + 3$$

$$p \text{ mod } 5 = \boxed{3}$$

4. Reorder the premises and state the conclusion that follows logically from the argument. (It may be helpful to rewrite some of the statements in if-then form and replace some statements with their contrapositives.

   1. If the table switches dealers, then the table becomes unlucky.
   2. If a table doesn't switch dealers, then I tend to win.
   3. If the table only has a \$10 minimum bet, I wait for an end seat.
   4. Every time I get a seat on the end, I am lucky.
   5. I don't win if there are complaining people at the table.

(3) If the table only has a \$10 minimum bet, I wait for an end seat.

(4) Every time I get a seat on the end, I am lucky.

(1) If the table switches dealers, then the table becomes unlucky.

Note: $S \to \sim L$ c.p. $L \to \sim S$

∴ The table doesn't switch dealers.

(2) If the table doesn't switch dealers, I tend to win.

(5) I don't win if there are complaining people at the table.

$\boxed{\text{∴ If there is a \$10 minimum bet and I get an end seat, then there are no complaining people at the table and I win.}}$

5. Use the following premise to determine whether the following arguments are valid or invalid (justify your answer accordingly):

For all positive integers n, if n is greater than 4, then n^2 is less than 2^n.

K: n is greater than 4

j: $n^2 < 2^n$

$K \rightarrow j$

a. Premise: $100 > 4$

Conclusion: $\therefore 100^2 < 2^{100}$

$K \rightarrow j$

K

$\therefore j$ modus ponens

$\boxed{\text{Valid.}}$

b. Premise: $1^2 < 2^1$

Conclusion: $\therefore 1 > 4$

$K \rightarrow j$

j

$\therefore K$

$\boxed{\text{Invalid. Converse Error.}}$

c. Premise: $3 \leq 4$

Conclusion: $\therefore 3^2 \geq 2^3$

$K \rightarrow j$

$\sim K$

$\therefore j$

$\boxed{\text{Invalid. Inverse Error.}}$

d. Premise: $2^2 \geq 2^2$

Conclusion: $\therefore 2 \leq 4$

$K \rightarrow j$

$\sim j$

$\therefore \sim K$ modus tollens

$\boxed{\text{Valid.}}$

e. Use symbols and notation to write the negation of the above premise:

For all positive integers n, if n is greater than 4, then n^2 is less than 2^n.

$\forall n \in \mathbb{Z}^+, n > 4 \rightarrow n^2 < 2^n$

negation: $\exists n \in \mathbb{Z}^+ \mid n \leq 4$ and $n^2 \geq 2^n$

6. Determine whether the statement is true or false, justifying your answer with a proof or counterexample as appropriate. (If true, use the definitions and properties of even and odd integers)

   a. For all integers n, if n is odd then $(-1)^n = -1$.

Let p be an odd integer.

$p = 2k+1,\ k \in \mathbb{Z}$

$(-1)^p = (-1)^{2k+1}$

$= \underbrace{(-1)^2 \cdot (-1)^2 \cdot \cdots \cdot (-1)^2}_{k\ (-1)^2 s} \cdot (-1)$

$= 1 \cdot 1 \cdot 1 \cdot \cdots \cdot 1 \cdot (-1)$

$= -1$

$\therefore \forall n \in \mathbb{Z}$, n is odd $\rightarrow (-1)^n = -1$.

   b. The product of any two even integers is divisible by 4.

Let k and j be even integers

Thus, $k = (2n)$ and $j = (2m)$, $n, m \in \mathbb{Z}$

$k \cdot j = (2n)(2m)$

$= 4(nm)$

$4 \mid 4nm$

$\therefore$ The product of any two even integers is divisible by 4.

   c. The number 0.123123123… is a rational number.

Let $p = 0.123123123\ldots$

$1000p = 123.123123123\ldots$

$1000p - p = 123$

$999p = 123$

$p = \frac{123}{999}$

$p = \frac{41}{333}$

$41, 333 \in \mathbb{Z}$

$\therefore p \in \mathbb{Q}$

d. For all integers $n > 1$, $n^2 + 2n + 5$ is a prime number.

Let $n = 5$:

$n^2 + 2n + 5$

$(5)^2 + 2(5) + 5$

40

$2 \mid 40$

40 is composite

$\boxed{\text{False.}}$

e. For all integers a, b, and c, if $if\ a \nmid bc,\ then\ a \nmid b$.

$\boxed{\text{True.}}$

f. For all integers n, the square of n is either of the form $3k\ or\ 3k + 1,\ k \in \mathbb{Z}$.

Let $n \in \mathbb{Z}$

$n^2 \bmod 3 \equiv (n \bmod 3)^2$

If $n \bmod 3 = 0$:

$0^2 = 0 = 3K$, $K =$ [illegible]

If $n \bmod 3 = 1$:

$1^2 = 1 = 3K + 1$, $K = 0$

If $n \bmod 3 = 2$:

$2^2 = 4$

$4 \bmod 3 = 1 = 3K + 1$, $K = 0$

7. Let x represent a student from the set of all UCF students, and let y represent a course from the set of courses offered at UCF. Given the following predicates:

$F(x)$ – "x is a freshman"; $T(x,y)$ – "x is taking y"; $P(x,y)$ – "x passed y"

Use quantifiers (single or multiple as needed) to express the following statements:

a. No student is taking every advanced course.

$\forall x, \exists y \mid y$ is an advanced course $\wedge \sim T(x,y)$

b. Every freshman passed calculus.

$\forall x,\ F(x) \rightarrow P(x, \text{calculus})$

c. Some advanced courses are being taken by no students.

$\forall y, \exists x \mid y$ is an advanced course $\wedge \sim T(x,y)$

d. Some freshmen are only taking advanced courses.

$\exists x \mid \forall y, F(x) \wedge T(x,y) \rightarrow y$ is an advanced course.

e. No freshman has taken and passed Differential Equations.

$\forall x,\ F(x) \wedge T(x, \text{DiffEQ}) \rightarrow \sim P(x, \text{DiffEQ})$

BONUS:

You meet eight inhabitants on an island of Knights and Knaves: Kim, Marie, Wendy, Aisha, Rachael, Willene, Anuradha and Harrison. Kim says, "Neither Aisha nor Wendy are knights." Marie says, "Anuradha and I are both knights." Wendy says that Harrison is a knave. Aisha tells you that Anuradha could claim that Harrison is a knight. Rachael says, "Willene is a knave." Willene says that neither Kim nor Harrison are knights. Anuradha claims, "Only a knave would say that Aisha is a knave." Harrison says that Aisha is a knave.

Can you determine who is a knight and who is a knave? (circle as appropriate)

Kim is a KNIGHT / KNAVE

Marie is a KNIGHT / KNAVE

Wendy is a KNIGHT / KNAVE

Aisha is a KNIGHT / KNAVE

Rachael is a KNIGHT / KNAVE

Willene is a KNIGHT / KNAVE

Anuradha is a KNIGHT / KNAVE

Harrison is a KNIGHT / KNAVE

ANSWERS:

1. 1
2. Saturday
3. $a)\ 2;\quad b)\ 3$
4. Re-ordered premises: 3, 4, 1(cp), 2, 5(cp);
   Conclusion: If $10 minimum & I get an end seat, then I win and there are no complaining people.
5. $a)\ VALID - universal\ modus\ ponens$
   $b)\ INVALID - converse\ error$
   $c)\ INVALID - inverse\ error$
   $d)\ VALID - universal\ modus\ tollens$
   $e)\ \exists n \in \mathbb{Z}^+ \mid (n > 4) \wedge (n^2 \geq 2^n)$
6. $a)\ TRUE;\quad b)\ TRUE;\quad c)\ TRUE;\quad d)\ FALSE;\quad e)\ TRUE;\quad f)\ TRUE$
7. $a)\ \forall x,\ \exists y \mid y\ is\ an\ advanced\ course \wedge {\sim}T(x,y)$
   $b)\ \forall x, F(x) \rightarrow P(x, calculus)$
   $c)\ \exists y \mid \forall x,\ \ y\ is\ an\ advanced\ course \wedge {\sim}T(x,y)$
   $d)\ \exists x \mid \forall y,\ \ F(x) \wedge T(x,y)\ \rightarrow y\ is\ an\ advanced\ course$
   $e)\ \forall x,\ \ F(x) \rightarrow [T(x, DiffEQ) \wedge {\sim}P(x, DiffEQ)]$

BONUS: Knights: Kim, Rachael, Harrison; Knights: Marie, Wendy, Aisha, Willene, Anuradha