Discrete Mathematics
Exam #4 (Chapter 9)
FALL 2012

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SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT!!!

1. Consider the positive integers between 1000 and 9999 inclusive. (Use the counting principles discussed in class to answer each of the following; show your justification for each part).

   a. How many positive integers are in the subset?

$$9999 - 1000 + 1 = \boxed{9000}$$

   b. How many are even?

$$\frac{(9998 - 1000)}{2} + 1 = \boxed{4500}$$

   c. How many are divisible by 9?

$$\frac{(9999 - 1008)}{9} + 1 = \boxed{1000}$$

   d. How many are divisible by 5 or 7?

$$\left(\frac{(9995 - 1000)}{5} + 1\right) + \left(\frac{(9996 - 1001)}{7} + 1\right) - \left(\frac{(9975 - 1015)}{35} + 1\right) = \boxed{2829}$$

2. A committee is formed consisting of one representative from each of the 50 states in the United States, where the representative from a state is either the governor or one of the two senators from that state. How many ways are there to form this committee?

$$\boxed{3^{50}}$$

3. Find the number of 5-permutations of a set with nine elements.

$$\frac{9!}{(9-5)!} = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = \boxed{15120}$$

4. Prove that for all integers $n \geq 3$, $P(n+1,\ 3) - P(n,\ 3) = 3\,P(n, 2)$

$$\frac{1}{n!}\left(\frac{(n+1)!}{(n-2)!} - \frac{n!}{(n-3)!}\right) = \left(3\frac{n!}{(n-2)!}\right)\frac{1}{n!} \quad \forall n \in \mathbb{Z} \mid n \geq 3$$

$$(n-2)!\left(\frac{n+1}{(n-2)!} - \frac{1}{(n-3)!}\right) = \left(\frac{3}{(n-2)!}\right)(n-2)!$$

$$n+1 - \frac{(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = 3$$

$$n+1-(n-2) = 3$$

$$n+1-n+2 = 3$$

$$3 = 3 \quad \checkmark$$

5. While serving 5-year sentences in prison for reckless driving charges, the ladies in 4th period spend their time making license plates.

a. Sienna is instructed to make license plates using either three digits followed by three letters or three letters followed by three digits. How many distinct license plates can she make?

$$10^3 26^3 + 26^3 10^3 = \boxed{35152000}$$

b. Thuy has to make license plates using either two letters followed by four digits or two digits followed by four letters. How many distinct license plates can she make?

$$26^2 10^4 + 10^2 26^4 = \boxed{52457600}$$

c. Rosemary's license plates are made using either four letters followed by two digits or four digits followed by two letters. How many distinct license plates can she make?

$$26^4 10^2 + 10^4 26^2 = \boxed{52457600}$$

d. Sam's license plates can have letters and digits in any order, each plate with six distinct characters (each character is used only once on each plate). How many distinct license plates can she make?

$$P(36, 6) = \frac{36!}{30!} = \boxed{1402410240}$$

6. In the Harry Potter books/movies, it was revealed that Tom Marvolo Riddle rearranged the letters in his name to spell out "I Am Lord Voldemort". He knew he wanted to rearrange his name to start with the phrase "I Am".

   a. How many ways could he have rearranged the letters in his name to form his alias?

len("LORDVOLDEMORT") = 13

2 3 2 2

$$\frac{13!}{2!3!2!2!} = \boxed{129729600}$$

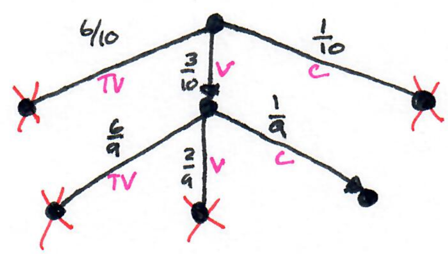
   b. How many ways could he have rearranged the letters in his name to form his alias, if he had kept the title "Lord"?

len("VOLDEMORT") = 9

2

$$\frac{9!}{2!} = \boxed{181440}$$

7. The host of a game show I drawing chips from a bag to determine the prizes for which contestants will play. Of the 10 chips in the bag, six are labeled "television", three are labeled "vacation", and one is labeled "car". If the host draws the chips at random and does not replace them, draw a probability tree and use your diagram to calculate the probability that he draws a vacation and then a car.



$$\frac{3}{10} \cdot \frac{1}{9} = \frac{3}{90} = \boxed{\frac{1}{30}}$$

ANSWERS:

1. $a.\ 9000$ $b.\ 4500$ $c.\ 1000$ $d.\ 1829$
2. 3^{50}
3. 15120
4. $HINT: substitute\ formulas\ for\ proof$
5. $a.\ 35152000$ $b.\ 52457600$ $c.\ 52457600$ $d.\ 1402410240$
6. $a. \frac{13!}{3!2!2!2!}$ $b. \frac{8!}{2!}$
7. $\frac{3}{90} = \frac{1}{30}$