Discrete Mathematics
Exam #4 (Chapter 9)
FALL 2013

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SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT!!!

1. Consider the positive integers between 100 and 99999 inclusive. (Use the counting principles discussed in class to answer each of the following; show your justification for each part).
   a. How many positive integers are in the subset?

   $$99999 - 100 + 1 = \boxed{99900}$$

   b. How many are divisible by 6?

   $$\frac{(99996 - 102)}{6} + 1 = \boxed{16650}$$

   c. How many consist of distinct digits?

   $$9 \cdot P(9,2) + 9 \cdot P(9,3) + 9 \cdot P(9,4)$$

   $$9 \cdot \left(\frac{9!}{7!} + \frac{9!}{6!} + \frac{9!}{5!} \right) = 9(9 \cdot 8 + 9 \cdot 8 \cdot 7 + 9 \cdot 8 \cdot 7 \cdot 6)$$

   $$= \boxed{32400}$$

2. Two six-sided dice are rolled, one blue and one red. What is the probability that either the blue die is 3 or the sum of the dice is even?

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$\frac{21}{36} = \boxed{\frac{7}{12}}$$

3. a. I play the lottery twice a week (Wednesdays and Saturdays), always playing the same six numbers. The Florida Lottery administrators randomly selects six balls; each is assigned a number from 1 through 53, and the balls that are drawn from random correspond to the winning numbers. Once a ball is selected, it is not returned to the selection bin. **How many combinations of lottery tickets could be purchased?**

$$\binom{53}{6} = \frac{53!}{6!\,47!} = \boxed{22957480}$$

b. I also play Powerball with the same frequency. The winning Powerball numbers are also comprised of six numbers; however, five are selected from one bin (balls corresponding to numbers 1 through 59 – again, once they are selected, they are not returned to the selection bin), and the sixth ball is selected from a different bin for Powerballs (the Powerballs are numbered 1 through 35). **What is the probability that I would win the Powerball with all six winning numbers (five numbers plus the Powerball number)?**

$$35 \cdot \binom{59}{5} = \frac{35 \cdot 59!}{5!\,54!} = 175223510$$

Probability: $\boxed{\dfrac{1}{175223510}}$

4. There are 29 eligible swimmers this year. If four of them can swim the 200 Medley Relay, how many ways can the relay be formed?

$$\binom{29}{4} = \frac{29!}{25!\,4!} = \boxed{23751}$$

5. Find the coefficient of the x^9y^{10} in $(2x - 3y^2)^{14}$ when the expression is expanded by the binomial theorem.

$$\binom{14}{5} \cdot 2^9 \cdot (-3)^5 = \boxed{-249080832}$$

6. An auto insurance company has 10,000 policyholders. Each policyholder is classified as:
   (1) Young or old;
   (2) Male or female;
   (3) Married or single.

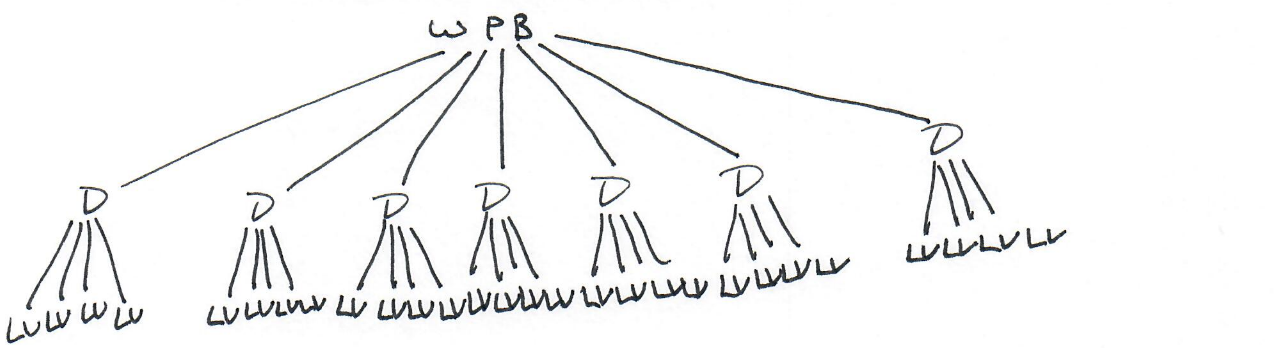
   Of these policyholders, 3000 are young, 4600 are male, and 7000 are married. The policyholders can also be classified as 1320 young males, 3010 married males, and 1400 young married persons. Finally, 600 of the policyholders are young married males. How many of the company's policyholders are young, female, and single?

3000 young → 7000 old
4600 male → 5400 female
7000 married → 3000 single

3000 young & 1320 young males → 1680 young females
7000 married & 3010 married males → 3990 married females
5400 female & 3990 married females → 1410 single females
3000 young & 1400 young married → 1600 young single
1400 young married & 600 young married males → 800 young married females
1680 young females & 800 young married females

→ $\boxed{\text{880 young, single females}}$

7. Suppose there are seven available flights from West Palm Beach to Dallas and four available flights from Dallas to Las Vegas. Construct a tree diagram to determine how many ways it is possible to travel from West Palm Beach to Las Vegas.



$\boxed{\text{28 ways}}$ (assuming no back tracking)

8. Prove that for all integers $n \geq 4$, $P(n+1,\ 4) - P(n,\ 4) = 4\,P(n,3)$

$$\frac{(n-3)!}{n!}\left(\frac{(n+1)!}{(n-3)!} - \frac{n!}{(n-4)!}\right) = \left(4\,\frac{n!}{(n-3)!}\right)\frac{(n-3)!}{n!}$$

$$\frac{(n+1)!}{n!} - \frac{(n-3)!}{(n-4)!} = 4$$

$$\frac{(n+1)\cancel{n!}}{\cancel{n!}} - \frac{(n-3)\cancel{(n-4)!}}{\cancel{(n-4)!}} = 4$$

$$n+1-(n-3) = 4$$

$$n+1-n+3 = 4$$

$$4 = 4 \quad \checkmark$$

BONUS: Change Fibonnaci's problem so that each pair of adult rabbits produces 2 pairs of rabbits per litter (as opposed to just one pair). Show the new sequence and use it to determine how many pairs of rabbits will be alive halfway through the 12th month?

	Fresh	Pregnant	Tot
month 0:	1	0	1
1:	0	1	1
2:	2	1	3
3:	2	3	5
4:	6	5	11
5:	10	11	21
6:	22	21	43

$a_0 = a_1 = 1$

$a_n = a_{n-1} + 2a_{n-2}$

$a_{11} = \boxed{1365 \text{ pairs}}$

1, 1, 3, 5, 11, 21, 43, 85, 171, 341, 683, 1365