Discrete Mathematics
Exam #4 (Chapter 9)
FALL 2014

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Period: 7

SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT!!!

1. Consider the set of even integers between 1 and 1000 inclusive. (Use the counting principles discussed in class to answer each of the following; show your justification for each part).
   a. How many are divisible by 6?

$$\frac{(996-6)}{6} + 1 = \boxed{166}$$

   b. How many consist of repeated digits?

$$\underbrace{0}_{\text{single digit}} + \underbrace{9}_{\substack{\text{double digit}\\(11,44,\text{etc.})}} + \left(\underbrace{\left(\frac{(999-100)}{1} + 1\right)}_{\text{all 3 digit \#}} - \underbrace{9 \cdot P(9,2)}_{\text{3 distinct digits}}\right) + \underbrace{1}_{1000} = \boxed{262}$$

2. Find the fifth term in $(4x^2 - y)^9$ when the expression is expanded by the binomial theorem.

$$\binom{9}{4} \cdot (4x^2)^5 \cdot (-y)^{9-5} = \boxed{129024\, x^{10} y^4}$$

3. Mr. Oddi is sitting at the blackjack table and is in luck – they are only playing with one standard deck (4 suits, 2-10 & JQKA) of cards!

   a. The dealer deals two cards to herself and to Mr. Oddi. Mr. Oddi is dealt the 10 of hearts and the 7 of clubs. The dealer only has one card showing – the 8 of diamonds. What is the probability that her hidden card is a 10, J, Q, K, or A?

$$\boxed{\frac{19}{49}}$$

   b. Her other card was the 10 of spades and she won (neither of the players drew a third card). The dealer puts the four used cards in the discard pile, and deals again. Mr. Oddi is now dealt the queen of hearts and the 10 of diamonds. The dealer has the Ace of clubs showing. What is the probability that her two cards have a value of 21 (i.e. her hidden card is a 10, J, Q, or K)?

2 4 3 4

$$\boxed{\frac{13}{45}}$$

4. A club has seven members. Three are to be chosen to go as a group to a national meeting.
   a. How many distinct groups of three can be chosen?

   $$\binom{7}{3} = \frac{7!}{3!\,4!} = \boxed{35}$$

   b. If the club contains four men and three women, how many distinct groups of three contain two men and one woman?

   $$\binom{4}{2}\binom{3}{1} = \boxed{18}$$

   c. If the club contains four men and three women, how many distinct groups of three contain at most two men?

   $$\binom{7}{3} - \binom{4}{3} = \boxed{31}$$

   d. If the club contains four men and three women, how many distinct groups of three contain at least one woman?

   $$\binom{7}{3} - \binom{4}{3} = \boxed{31}$$

   e. If the club contains four men and three women, what is the probability that a distinct group of three will contain at least one woman?

   $$\boxed{\frac{31}{35}}$$

   f. If two members of the club refuse to travel together as part of the group (but each is willing to go if the other does not), how many distinct groups of three can be chosen?

   $$35 - \binom{5}{1} = \boxed{30}$$

   g. If two members of the club insist on either traveling together or not going at all, how many distinct groups of three can be chosen?

   $$\binom{5}{1} + \binom{5}{3} = \boxed{15}$$

5. A large pile of coins consists of pennies, nickels, dimes, and quarters (at least 20 of each).
   a. How many different collections of 20 coins can be chosen?

$$\binom{20+4-1}{20} = \binom{23}{20} = \boxed{1771}$$

   b. How many different collections of 20 coins chosen at random will contain at least 3 coins of each type?

$$\binom{8+4-1}{8} = \binom{11}{8} = \boxed{165}$$

   c. What is the probability that a collection of 20 coins chosen at random will contain at least 3 coins of each type?

$$\frac{165}{1771} = \boxed{\frac{15}{161}}$$

6. For each of the following words, respond as directed:
   a. How many ways can the letters of the word COMPUTER be selected and written in a row?

$$8! = \boxed{40320}$$

   b. How many 5-permutations can be formed from the word VIRTUAL?

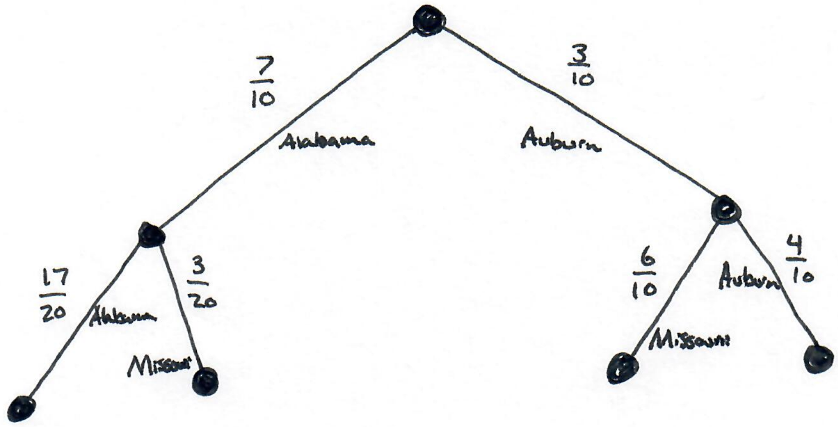
$$\frac{7!}{2!} = \boxed{2520}$$

   c. How many distinguishable ways can the letters of JAMESWILLIAMKRUMENACKER be arranged in order?

3 3 3 2 2 2 2

$$\frac{23!}{3!3!3!2!2!2!2!} = \boxed{7480328917501440000}$$

7. Auburn's football team is playing Alabama this weekend in their annual Iron Bowl. Analysts have determined that there is a 70% chance that Alabama will win, 30% chance that Auburn will win. The winner of this game will advance to the 2013 SEC Championship game to play against Missouri. The same analysts have determined that if they end up playing Auburn, Missouri has a 60% chance of winning the Championship, while if they play Alabama, Missouri will only have a 15% chance of winning the Championship. Draw a possibility tree for this scenario and use it to determine the probability of Missouri winning the SEC Championship game.



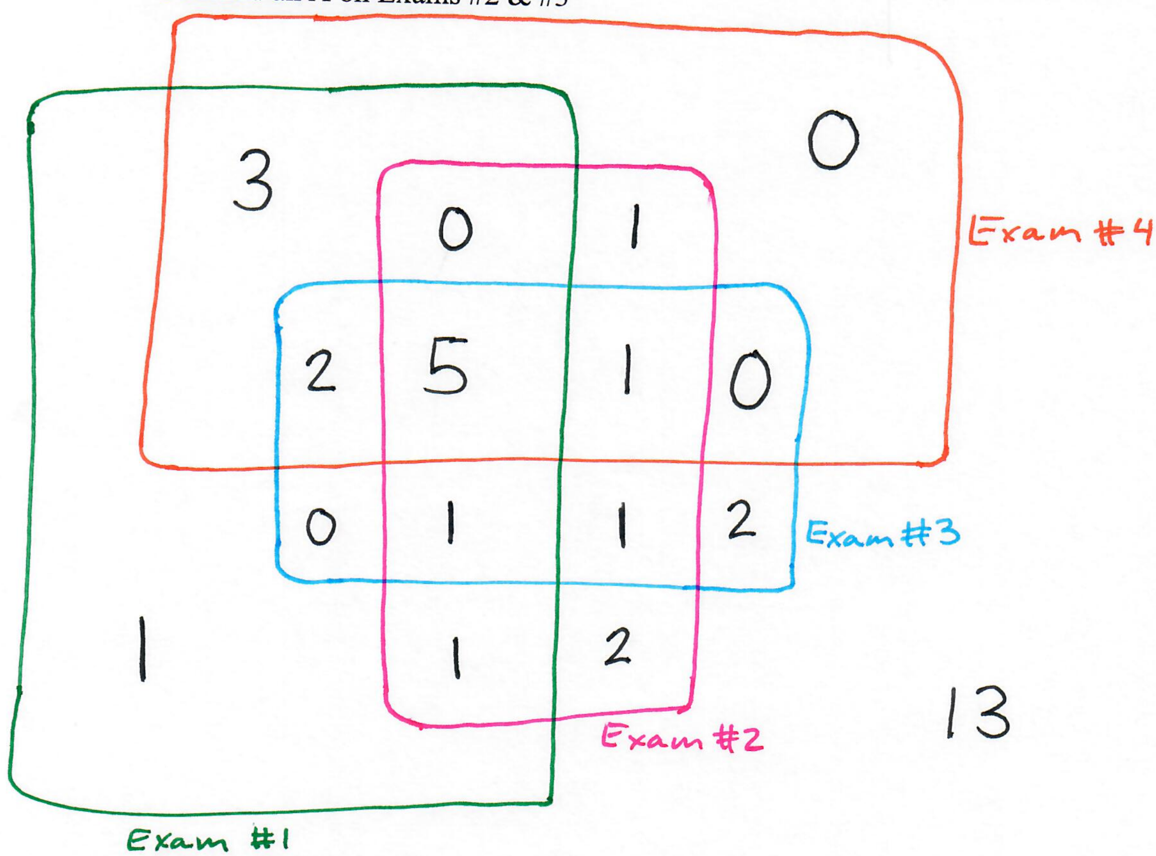
$$\frac{7}{10}\cdot\frac{3}{20} + \frac{3}{10}\cdot\frac{6}{10}$$

$$\frac{21}{200} + \frac{\overset{36}{\cancel{18}}}{\underset{200}{\cancel{100}}}$$

$$\boxed{\frac{57}{200}}$$

8. Last year's Discrete Math class had four exams. Of the 33 students in the class:

13 earned an A on Exam #1
12 earned an A on Exam #2
12 earned an A on Exam #3
12 earned an A on Exam #4
7 earned an A on Exams #1 & #2
8 earned an A on Exams #1 & #3
10 earned an A on Exams #1 & #4
8 earned an A on Exams #2 & #3
7 earned an A on Exams #2 & #4
8 earned an A on Exams #3 & #4
6 earned an A on Exams #1, #2, & #3
5 earned an A on Exams #1, #2, & #4
7 earned an A on Exams #1, #3, & #4
6 earned an A on Exams #2, #3, & #4
5 earned an A on all four exams



a. How many students never earned an A on an exam? 13

b. How many students earned an A on at least one exam? 20

c. How many students earned an A on Exam #3 only? 2

d. How many students earned an A on only Exams #1 & #2? 1

e. How many students in the class did not earn an A on Exam #4? 21