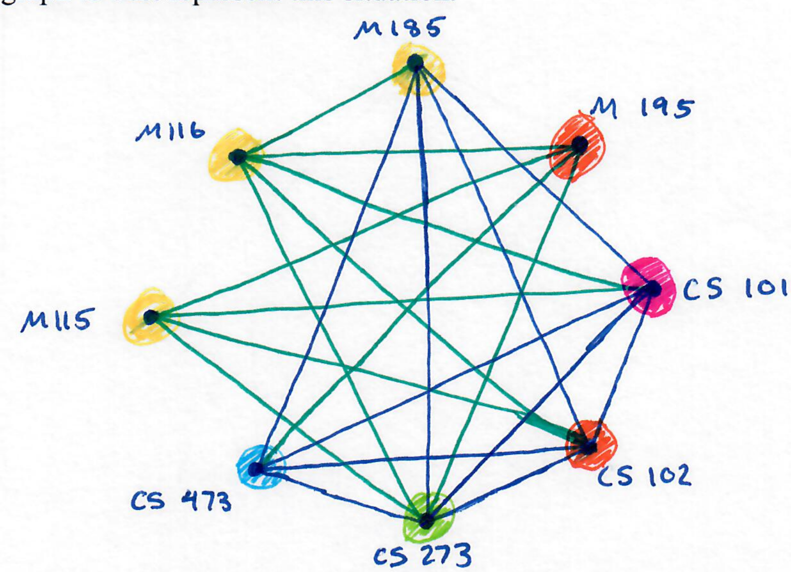
Graph Theory
Exam #1 (Chapter 1)
SPRING 2015

Name: Jacob Magnuson   Date: 2017-04-17   Period: 7

SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT!!!

1. You are tasked with scheduling the final exams for Math 115, Math 116, Math 185, Math 195, CS 101, CS 102, CS 273, and CS 473, using the fewest number of different time slots. If there are no students taking both Math 115 and CS 473, both Math 116 and CS 473, both Math 195 and CS 101, both Math 195 and CS 102, both Math 115 and Math 116, both Math 115 and Math 185, and both Math 185 and Math 195, but there are students in every other pair of courses.

   a. Draw a graph to best represent this situation.



   b. What is the fewest number of different time slots needed to administer all of the final exams?

$\boxed{5}$

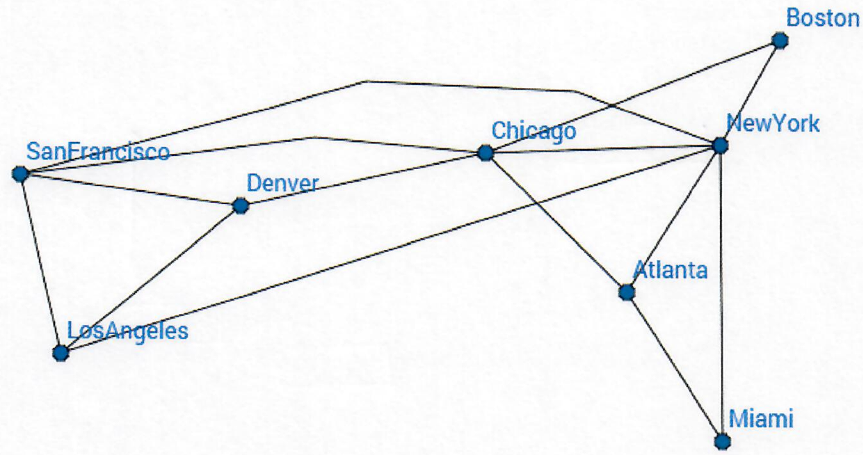
Slot 1	2	3	4	5
Math 115 Math 116 Math 185	Math 195 CS 102	CS 101	CS 273	CS 473

   c. What aspect of graph theory does this problem relate to? (Name the topic and state your answer to part (b) using a mathematical way to reference your answer.)

Graph Coloring — the graph's $\boxed{\text{chromatic number}}$ is 5.

2. The graph below illustrates travel availability between various cities in the United States. The table provides flight times of each flight and associated costs.

Atlanta – Chicago	1:40	$99
Atlanta – Miami	1:30	$69
Atlanta – New York	1:55	$79
Boston – Chicago	2:10	$79
Chicago – New York	1:50	$59
New York – San Francisco	4:05	$129
Chicago – San Francisco	2:55	$99
Denver – San Francisco	1:40	$79
Denver – Chicago	2:10	$69
Los Angeles – Denver	2:00	$89
Los Angeles – New York	3:50	$129
Miami – New York	2:45	$99
New York – Boston	0:50	$39
San Francisco – Los Angeles	1:15	$39

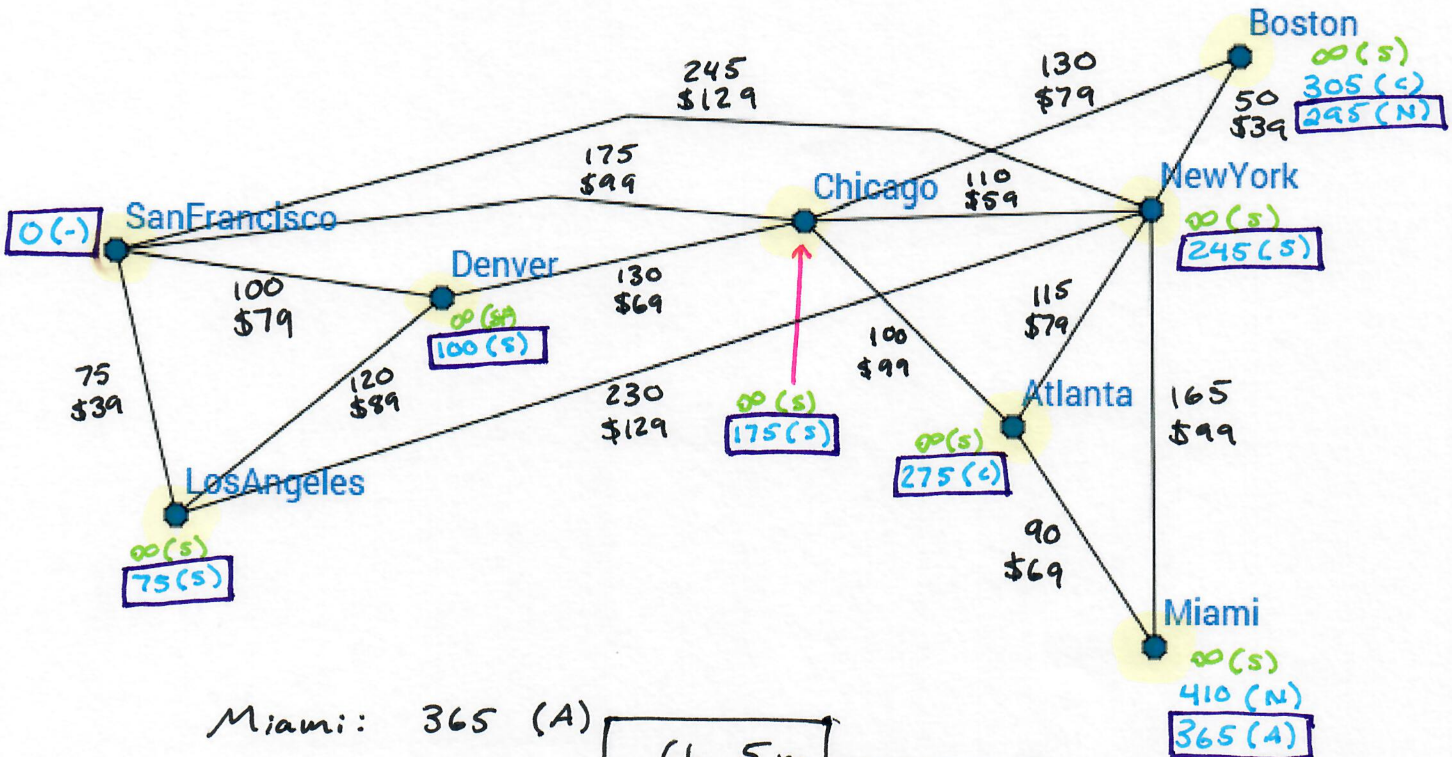


a. Use Dijkstra's to find the quickest path from San Francisco to Miami. (Provide the quickest path, the time of your path, and **permanent labels only** needed for this path.)

See next page.

b. Use Dijkstra's to find the cheapest path from San Francisco to Miami. (Provide the cheapest path, the cost of your path, and **permanent labels only** needed for this path.)

See next page.

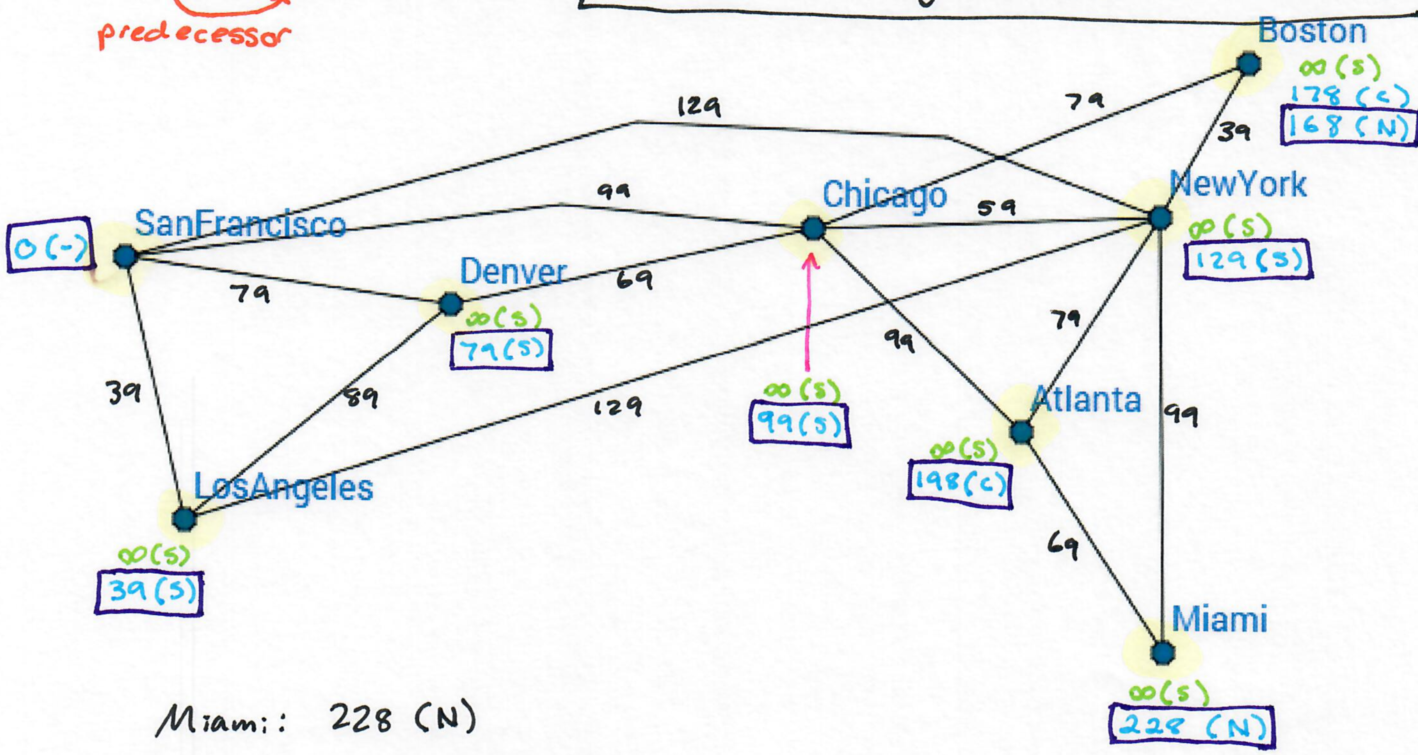


Miami:: 365 (A)

$M \leftarrow A \leftarrow C \leftarrow S$

predecessor

6h 5m
San Fran. → Chicago → Atlanta → Miami



Miami:: 228 (N)

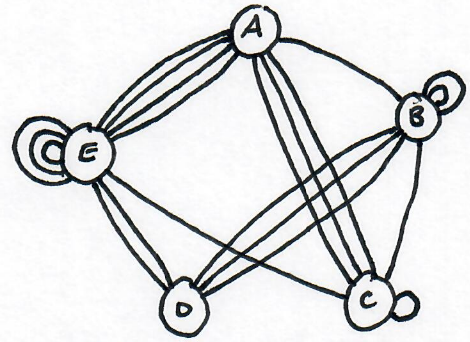
$M \leftarrow N \leftarrow S$

$228
San Fran. → New York → Miami

3. Given the following adjacency matrix:

$$\begin{array}{c} v_A \\ v_B \\ v_C \\ v_D \\ v_E \end{array}\begin{bmatrix} 0 & 1 & 3 & 0 & 4 \\ 1 & 2 & 1 & 3 & 0 \\ 3 & 1 & 1 & 0 & 1 \\ 0 & 3 & 0 & 0 & 2 \\ 4 & 0 & 1 & 2 & 3 \end{bmatrix}$$

a. Construct the corresponding multigraph.



b. How many paths of length 8 are there from vertex A to vertex D?

Adjacency Matrix $\rightarrow A$ $\quad A^{8}_{(0,3)} = \boxed{1,873,364}$

c. How many total paths of length 4 exist in the graph?

$$[1,1,1,1,1]\, A^{4} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = [16,048] \rightarrow \boxed{16048}$$

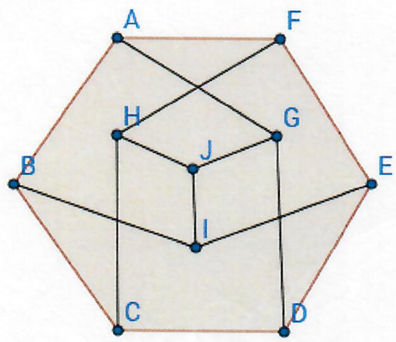
d. Classify the following paths:

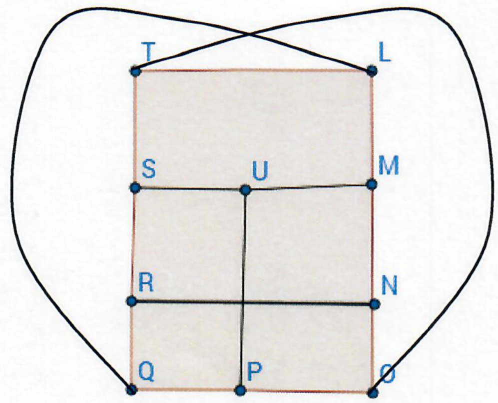
i. AEECBBA – $\boxed{\text{Circuit}}$

ii. DBABC – $\boxed{\text{walk}}$

4. For the following pairs of graphs, state whether the any of the graphs are isomorphic to each other. If they are isomorphic, justify the isomorphism. If they are not isomorphic to any of the others, state why this is the case.

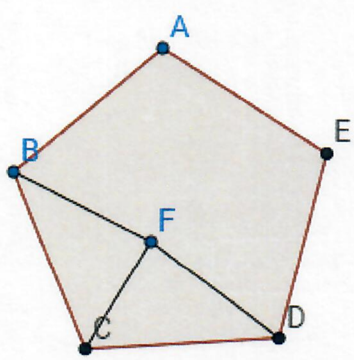
a.

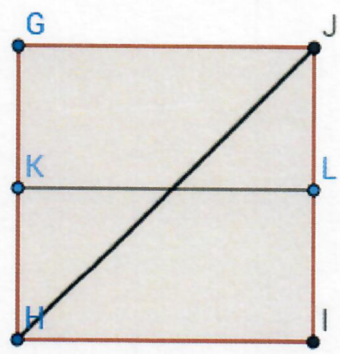




Isomorphic — A → M, B → U, C → P, D → O, E → T, F → L, G → N, H → Q, I → S, J → R

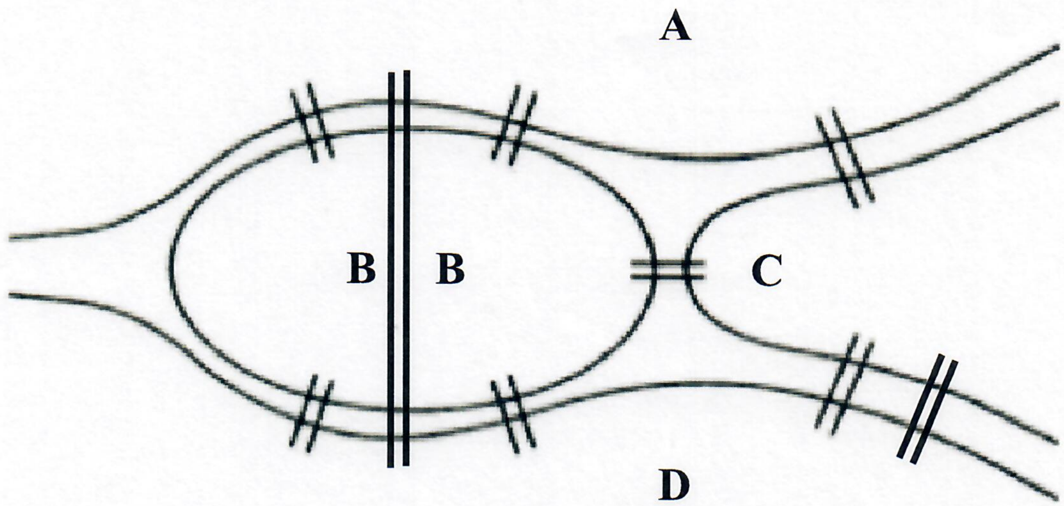
b.





Not Isomorphic — vertices with degree 2 are adjacent on left (A, E) but not on right (G, I)

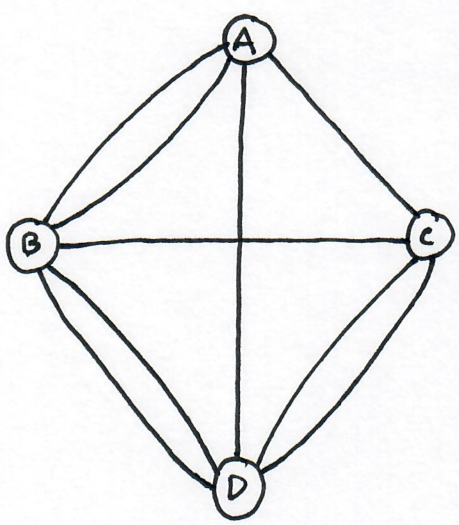
5. Consider the modified Königsberg problem of seven bridges to include two additional bridges (as shown below).



a. Suppose you are tasked with determining if someone could cross all nine of these bridges exactly once and return to the starting point. What **specific** type of path studied in this course does this problem represent?

$\boxed{\text{Eulerian Path}}$

b. Construct a graph to model this problem, and use the appropriate method to solve the task stated in part (a).

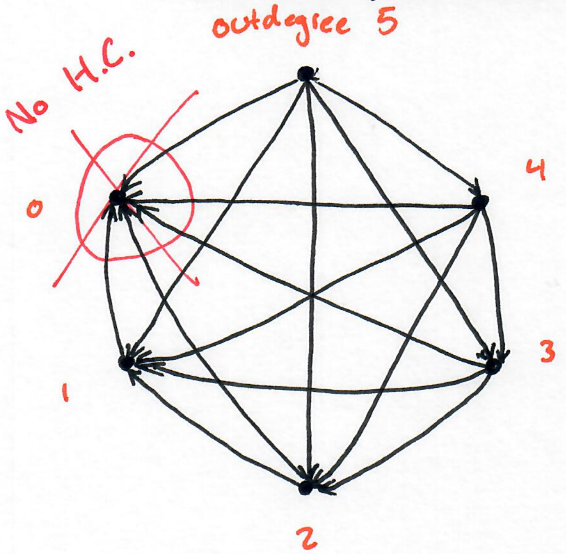


The path does not exist.

Vertices B and D are of odd degree.

6. A round-robin tournament is a tournament where each team plays every other team exactly once and no ties are allowed.

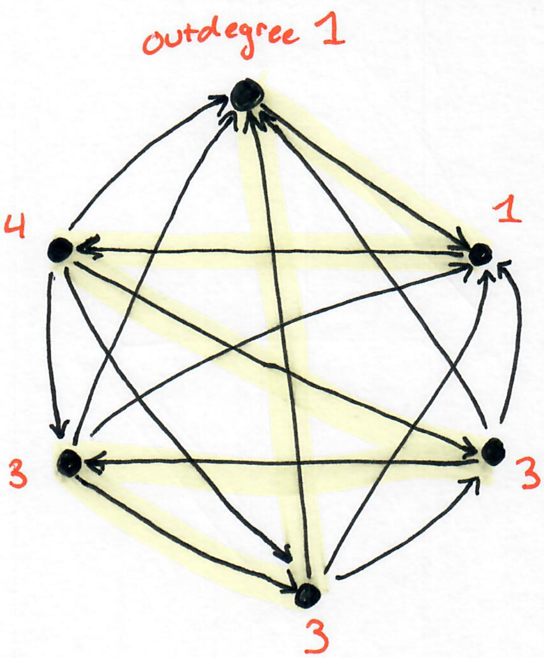
   a. Construct a graph to model the results of a round-robin tournament with six teams. Can a Hamiltonian cycle be found? Explain why or why not.



"No ties" means all vertices must have different # of wins (outdegree)

Ham. cycle DNE because ∴ a vertex with outdegree zero.

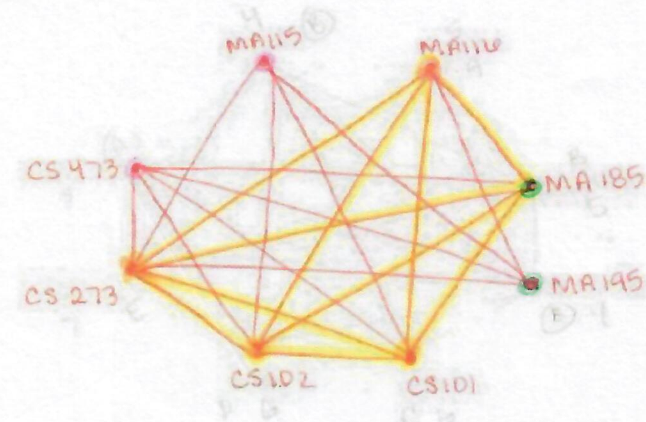
   b. Modify your answer for part (a) to allow two teams to tie with equal ranking. Can a Hamiltonian cycle be found under these conditions? Explain why or why not.



Yes. One such example is highlighted.

ANSWERS:

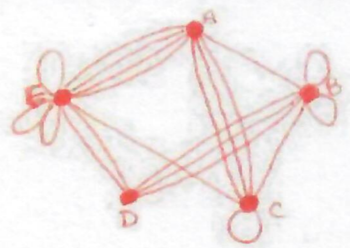
1. a. d



   b. 5 time slots; c. coloring (chromatic number of graph: n = 5)

2. a. San Francsico; Chicago; Atlanta; Miami – 6:05
   b. San Francisco; New York; Miami - $228

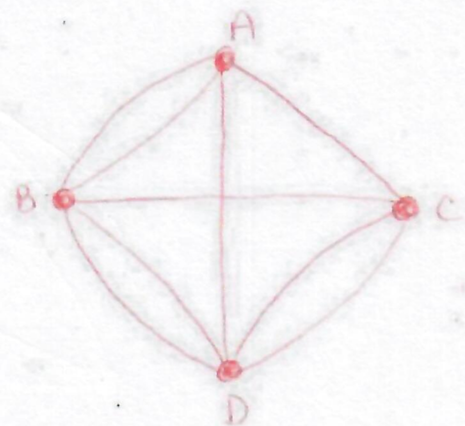
3. a.



   b. 1,873,364; c. 16,048; d. i) circuit ii) walk

4. a. isomorphic (Answers may vary; sample bijection: A=T, F=O, G=L, B=S, E=N, H=P, J=Q, C=U, D=M, I=R)
   b. not isomorphic (same degree sequence; but two vertices with degree 2 are adjacent on left and not on right)

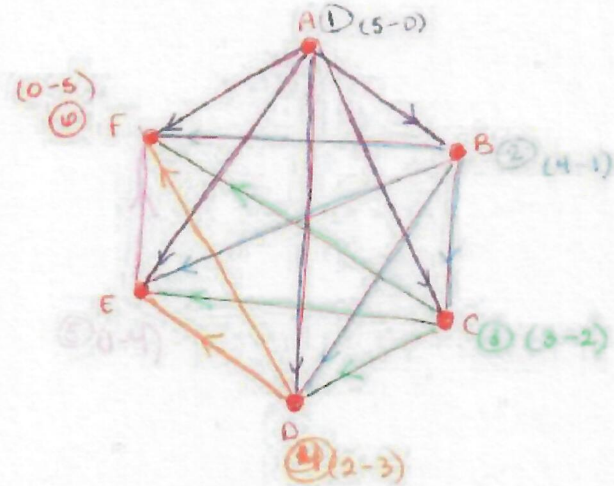
5. a. Euler circuit



   b. B & D have odd degree; therefore no Euler circuit exists

6. A round-robin tournament is a tournament where each team plays every other team exactly once and no ties are allowed.

   a. Construct a graph to model the results of a round-robin tournament with six teams. Can a Hamiltonian cycle be found? Explain why or why not.



~~Yes~~

~~· if directed~~ not Ham. cycle

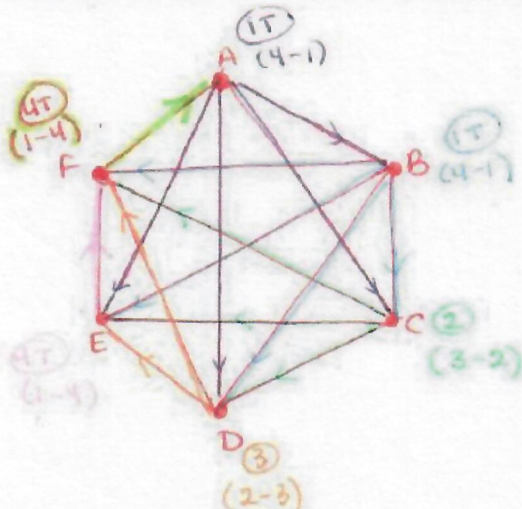
No

· if directed Ham. cycle

- one team lost all games

∴ no outdegree ⇒ ∴ no cycle

   b. Modify your answer for part (a) to allow two teams to tie with equal ranking. Can a Hamiltonian cycle be found under these conditions? Explain why or why not.



reverse A→F to F→A

now 2 teams tie w/ equal ranking

if not directed, yes

if directed, yes