Graph Theory
Exam #2 (Chapter 2)
SPRING 2014

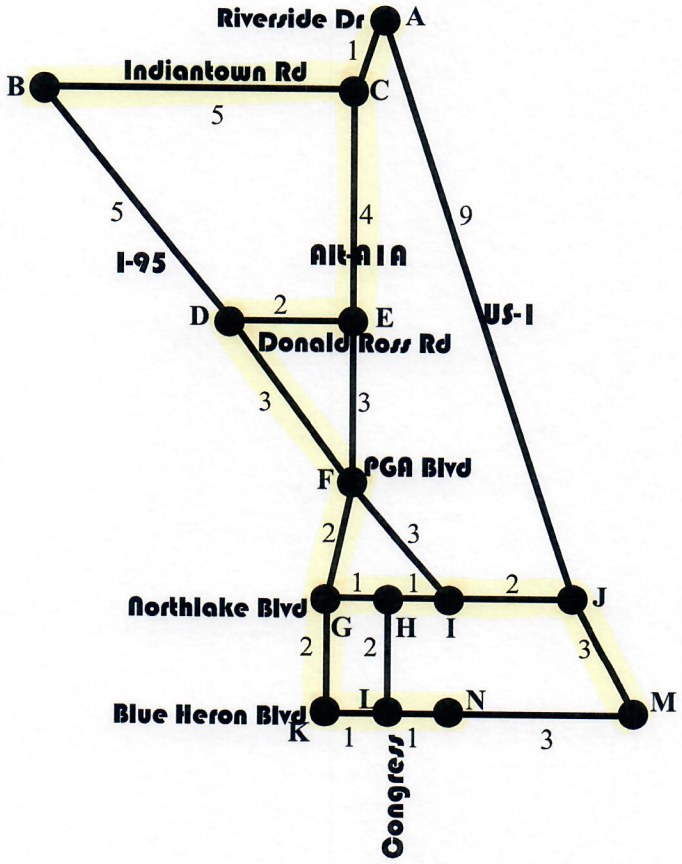
Jacob Magnuson
Name

2017-04-17
Date

7
Period

SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT!!!

1. Palm Beach County is working on implementing a system where traffic lights are communicating with one another, and to accomplish this they need to be linked. There is a debate in their office as to whether it would be more cost effective to predominantly use larger or smaller roadways to achieve this.
   a. They decide to focus their efforts first on using smaller roads. Use Kruskal's algorithm to find a minimal spanning tree. If there is a choice of edges to choose from, choose the edge that appears first in alphabetical order. (i.e. edge EF is before edge IJ). Draw the minimal spanning tree, or highlight the edges to be used on the diagram below.



Order Added

-AC
-GH
-HI
-KL
-LN
} 1's

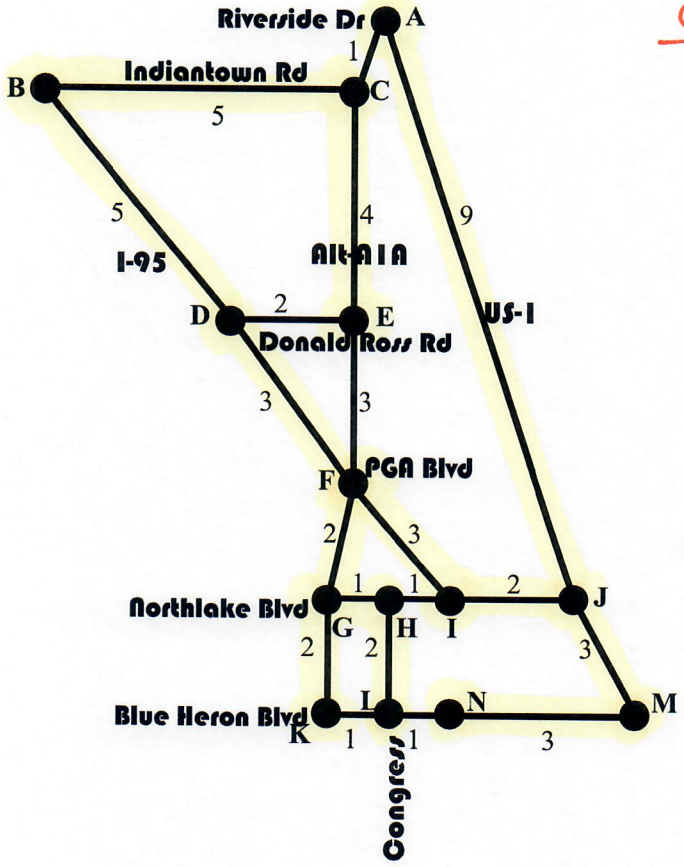
-DE
-FG
-GK
-IJ
} 2's

-DF
-JM
} 3's

-CE
} 4

-BC
} 5

b. The agency then decides to look at what happens using larger roads. Use Prim's algorithm to find a maximal spanning tree. If there is a choice of edges to choose from, choose the edge that appears first in alphabetical order. (i.e. edge EF is before edge IJ). Draw the maximal spanning tree, or highlight the edges to be used on the diagram below.

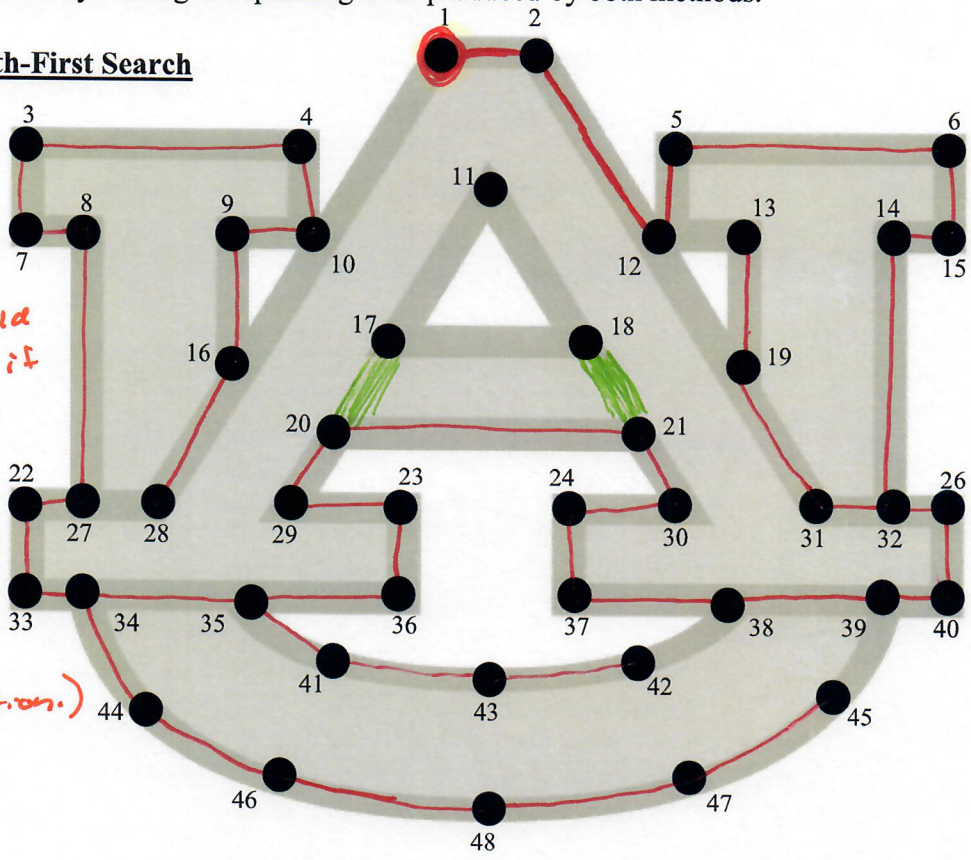


Order Added

- AJ
- JM
- MN
- IJ
- FI
- DF
- BD
- BC
- CE
- FG
- GK
- GH
- HL

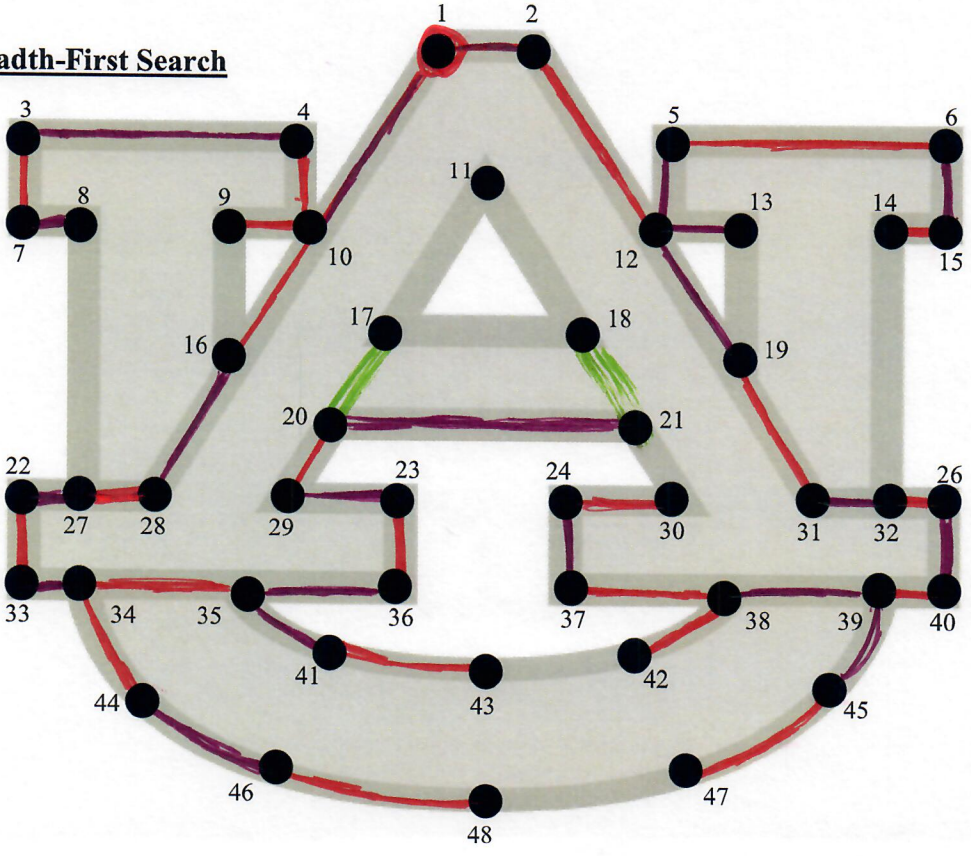
2. Demonstrate that you know the difference between the depth-first and breadth-first search algorithms by finding the spanning trees produced by both methods.

Depth-First Search



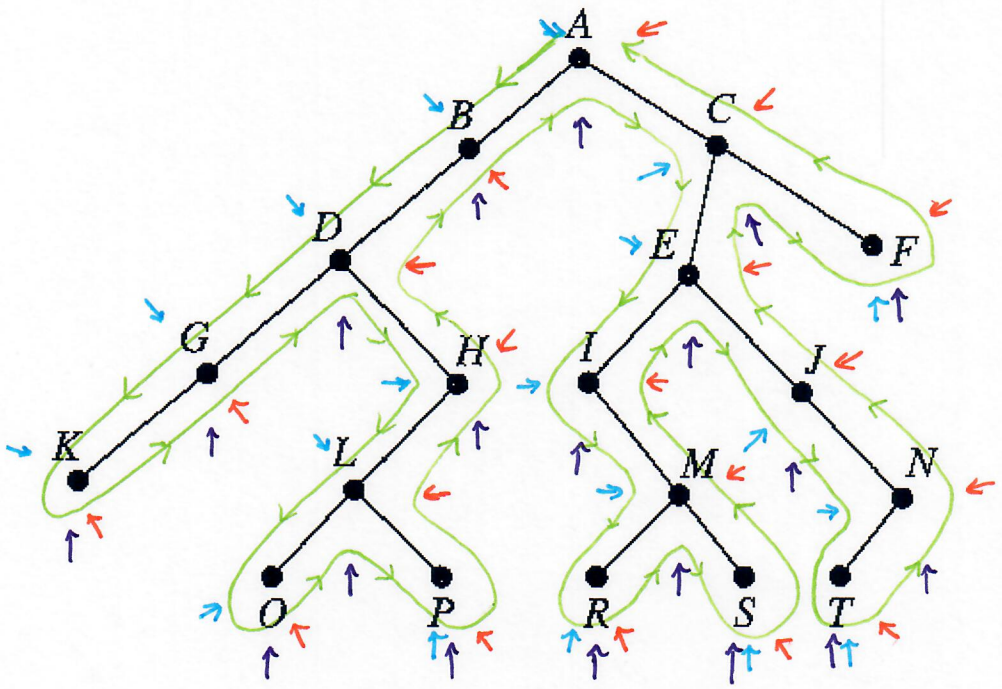
(Note: Oddi's answers would be produced if green edges existed — I assume this was clarified during this test's administration.)

Breadth-First Search



(see above)

3. Identify the designated listings of vertices for the following binary tree:

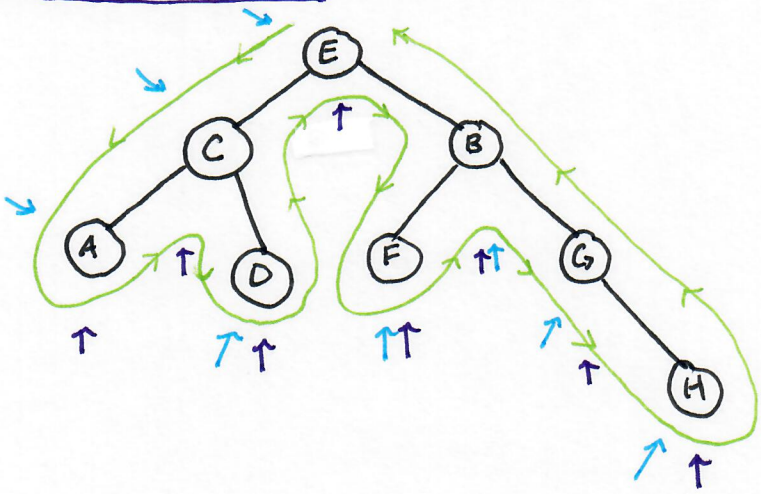


(as soon as it's seen) i. Preorder – A B D G K H L O P C E I M R S J N T F

(when you move back to its predecessor) ii. Postorder – K G O P L H D B R S M I T N J E F C A

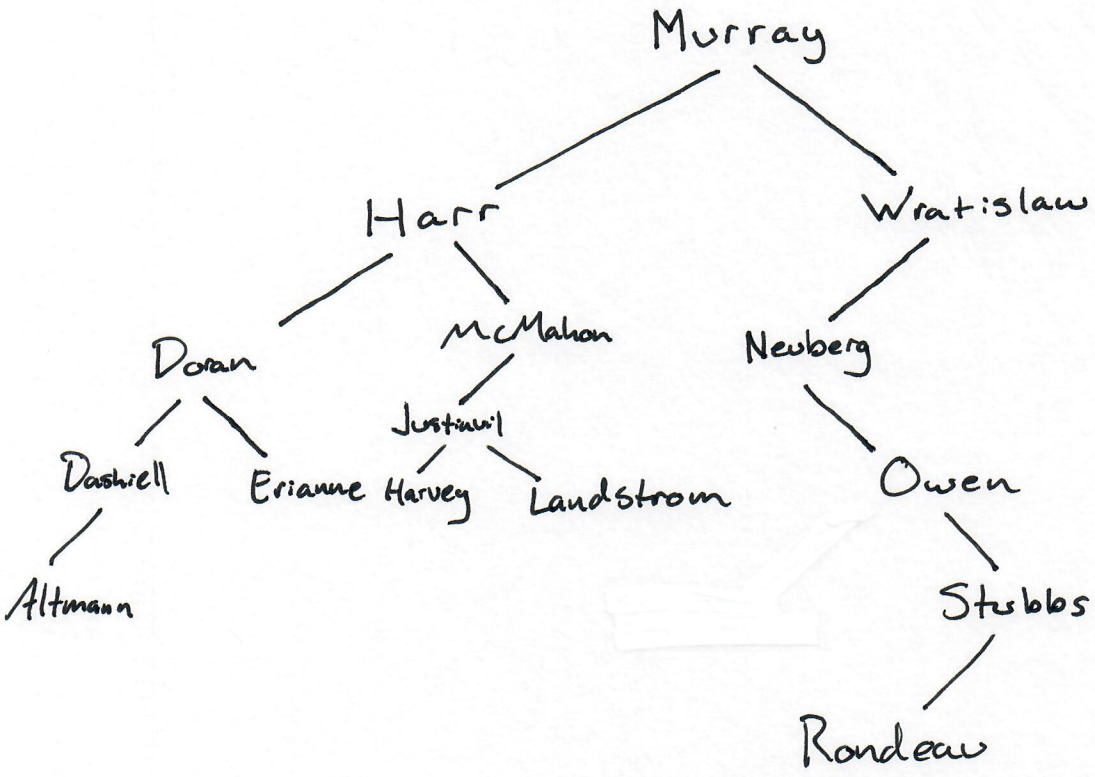
(between left and right subtrees) iii. Inorder – K G D O L P H B A I R M S E J T N C F

4. Construct a binary tree for which the preorder listing of vertices is E, C, A, D, B, F, G, H and the inorder listing is A, C, D, E, F, B, G, H.



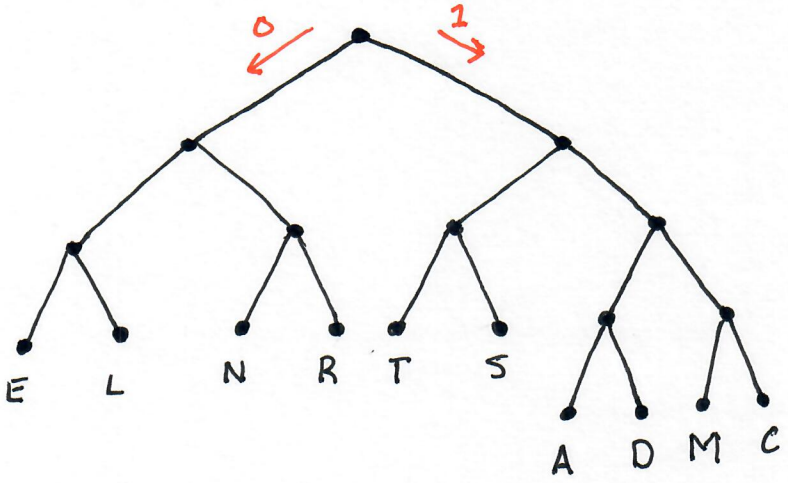
5. The science teachers department at Suncoast were hired in the following order: Murray, Wratislaw, Neuberg, Harr, Doran, Erianne, McMahon, Justinvil, Dashiell, Altmann, Owen, Harvey, Stubbs, Rondeau, Landstrom. Assuming Dr. Raiford's records of these teachers were created as they were hired, what would her binary search tree for the science department look like?

(Names alphabetically before the current name are placed on the left, and vice versa.)



6. The most commonly used letters in the English language are shown below with their expected usage rates. Find an optimal binary tree to assign codes of shortest length to each character. (In the construction of the tree, select a vertex with more children in preference to a vertex without children, and use as the left child the vertex of smaller weight or the vertex with more children if the two vertices have equal weight.)

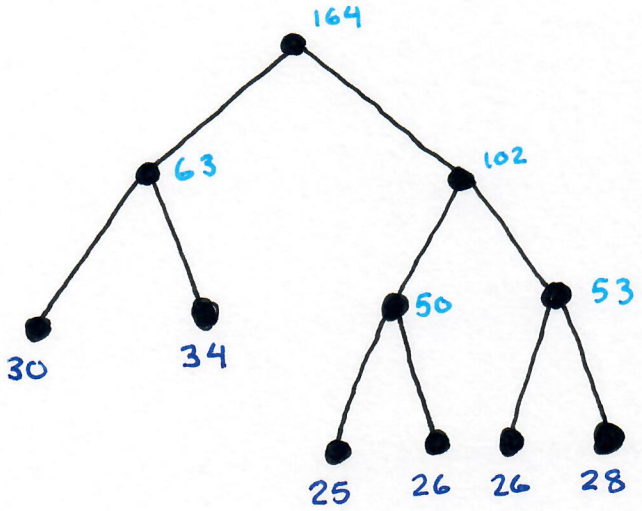
Letter	Usage	ASSIGNED CODEWORD
S	78	101
T	74	100
R	68	011
N	63	010
L	62	001
E	61	000
C	58	1111
M	56	1110
D	52	1101
A	51	1100



BONUS: How many levels of an optimal binary tree would be needed to assign codes of shortest length to each letter in the alphabet?

root → $1 + \lceil \log_2 26 \rceil = 6$

7. On the first day of school, Mr. Oddi has every student fill out an information sheet, which he files alphabetically in a binder. The number of students in each class is 26, 26, 25, 28, 30, and 34. Construct a binary tree to find the optimal merge pattern to merge these papers. Indicate the order in which they should be sorted, and the number of comparisons made.



When merging pre-sorted lists of size n and m, n+m-1 comparisons will be made. The -1 occurs when one item (from either list) is remaining; it goes straight to the back of the new list.

21
164
102
63
50
+ 53
432

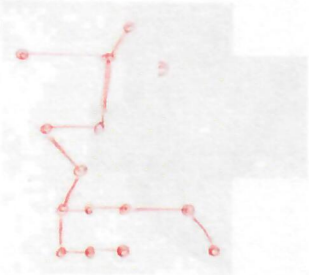
432 total comparisons

BONUS: Why is this not necessarily accurate, specific to Mr. Oddi's classes, of what occurred? ☺

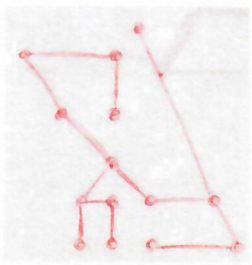
Some students may be in multiple classes.

ANSWERS:

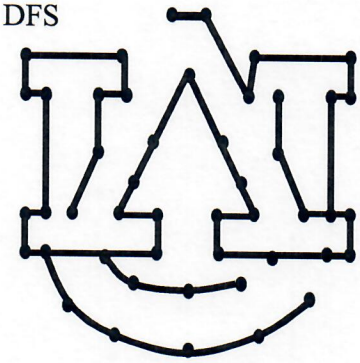
1. *a)*



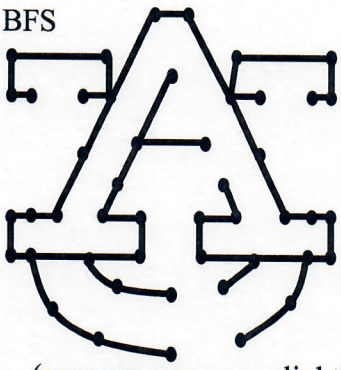
b)



2. DFS



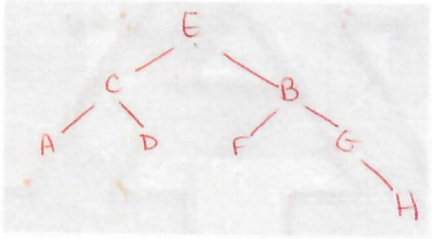
BFS



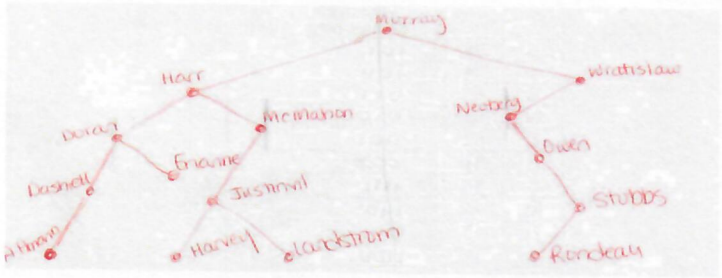
(answers may vary slightly)

3. i. ABDGKHLOPCEIMRSJNTF
   ii. KGOPLHDBRSMITNJEFCA
   iii. KGDOLPHBAIRMSEJTNCF

4.



5.



6.

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D	52	1101
A	51	1100

BONUS: 6 levels (counting root as 0^{th} level)

7. 25, 26, 26, 28, 30, & 34; 438 total comparisons
   BONUS: some students would be counted twice using those numbers