2.1 Logical Form and Logical Equivalence

p. 37 # 4, 7-10, 13, 15, 17, 21-22, 24, 33, 35, 37, 41-44, 50, 54

4. a) If p, then q. ;
   If q, then r. ;
   $\therefore$ If p, then r.

   b) a polynomial;
   differentiable;
   is continuous

7. $m \wedge \sim c$

8. a) $(h \wedge w) \wedge \sim s$
   b) $\sim w \wedge (h \wedge s)$
   c) $\sim (h \vee w \vee s)$
   d) $\sim (w \vee s) \wedge h$
   e) $w \wedge \sim (h \wedge s)$

9. $(n \vee k) \wedge \sim (n \wedge k)$
   Alternatively, $n \oplus k$

10. a) $(p \wedge q) \wedge r$
    b) $p \wedge \sim q$
    c) $p \wedge \sim (q \vee r)$
    d) $\sim p \wedge q \wedge \sim r$
    e) $\sim p \vee (q \wedge r)$

13.

p	q	$\sim(p \wedge q)$	$p \vee q$	$\sim(p \wedge q) \vee (p \vee q)$
T	T	F	T	T
T	F	T	T	T
F	T	T	T	T
F	F	T	F	T

15.

p	q	r	$\sim q$	$\sim q \vee r$	$p \wedge (\sim q \vee r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	F	F	F
F	F	T	T	T	F
F	F	F	T	T	F

17.

p	q	$\sim(p \wedge q)$	$\sim p \wedge \sim q$
T	T	F	F
T	F	T	F
F	T	T	F
F	F	T	T

$\sim(p \wedge q)$ and $\sim p \wedge \sim q$ do not have identical truth tables, so therefore they are not logically equivalent.

21.

p	q	r	$(p \wedge q) \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

$(p \wedge q) \wedge r$ and $p \wedge (q \wedge r)$ have identical truth tables, so therefore they are logically equivalent.

22.

p	q	r	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

$p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ have identical truth tables, so therefore they are logically equivalent.

(24)

p	q	r	$(p \vee q) \vee (p \wedge r)$	$(p \vee q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F

$(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$ do not have the same truth table, so therefore they are not logically equivalent.

(33) $x \leq -10 \vee 2 \leq x$

(35) $-1 < x \leq 1$

(37) $x < -7 \vee 0 \leq x$

(41)

p	q	$p \wedge \sim q$	$\sim p \vee q$	$(p \wedge \sim q) \wedge (\sim p \vee q)$
T	T	F	T	F
T	F	T	F	F
F	T	F	T	F
F	F	F	T	F

The statement forms a contradiction.

(42)

p	q	r	$\sim p \wedge q$	$q \wedge r$	$\wedge$	$\sim q$	$((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$
T	T	T	F	T	F	F	F
T	T	F	F	F	F	F	F
T	F	T	F	F	F	T	F
T	F	F	F	F	F	T	F
F	T	T	T	T	T	F	F
F	T	F	T	F	F	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	T	F

The statement forms a contradiction

(43)

p	q	$\sim p \vee q$	$p \wedge \sim q$	$(\sim p \vee q) \vee (p \wedge \sim q)$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

The statement forms a tautology.

(44) a) $(-\infty, 2) \vee \sim(1, 3)$

equiv. to $\sim[2, 3)$

b) $(-\infty, 1] \vee (-\infty, 2) \vee [3, \infty)$

equiv. to $\sim[2, 3)$

The statements are logically equivalent.

(50)

$(p \wedge \sim q) \vee p$	commutative; sub q for $\sim q$
$p \vee (p \wedge q)$	absorbtion
p	

(54)

$(p \wedge (\sim(\sim p \vee q))) \vee (p \wedge q)$	De Morgan's
$(p \wedge (\sim\sim p \wedge \sim q)) \vee (p \wedge q)$	Double Negative
$(p \wedge (p \wedge \sim q)) \vee (p \wedge q)$	Associative
$((p \wedge p) \wedge \sim q) \vee (p \wedge q)$	Idempotent
$(p \wedge \sim q) \vee (p \wedge q)$	Distributive
$p \wedge (\sim q \vee q)$	Negation
$p \wedge \mathbf{t}$	Identity
p	