2.2 Conditional Statements

p. 49 #1-4, 6-12, 13b, 14b, 17-19, 20bceg, 22bceg, 25, 30, 32-33, 41-42, 45, 50

(1) If a loop does not contain a stop or a go to, it will repeat exactly N times.

(2) If I catch the 8:05 bus, then I am on time for work.

(3) If you don't freeze, then I'll shoot.

(4) If you don't fix my ceiling, then I won't pay my rent.

(6)

p	q	$p \vee q$	$\sim p \wedge q$	$(p \vee q) \vee (\sim p \wedge q)$	$(p \vee q) \vee (\sim p \wedge q) \to q$
T	T	T	F	T	T
T	F	T	F	T	F
F	T	T	T	T	T
F	F	F	F	F	T

(7)

p	q	r	$p \wedge \sim q$	$p \wedge \sim q \to r$
T	T	T	F	T
T	T	F	F	T
T	F	T	T	T
T	F	F	T	F
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

(8)

p	q	r	$\sim p \vee q$	$\sim p \vee q \to r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	F
F	F	T	T	T
F	F	F	T	F

(9)

p	q	r	$p \wedge \sim r$	$q \vee r$	$(p \wedge \sim r) \leftrightarrow (q \vee r)$
T	T	T	F	T	F
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	T	F	F
F	T	T	F	T	F
F	T	F	F	T	F
F	F	T	F	T	F
F	F	F	F	F	T

(10)

p	q	r	$p \to r$	$q \to r$	$(p \to r) \leftrightarrow (q \to r)$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

(11)

p	q	r	$q \to r$	$p \wedge q$	$p \to (q \to r)$	$(p \wedge q) \to r$	$(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	T
T	F	T	T	F	T	T	T
T	F	F	T	F	T	T	T
F	T	T	T	F	T	T	T
F	T	F	F	F	T	T	T
F	F	T	T	F	T	T	T
F	F	F	T	F	T	T	T

(12) For any real number x, if $x > 2$, then $x^2 > 4$, and if $x < -2$, then $x^2 > 4$.

(13b)

p	q	p → q	~(p → q)	p ∧ ~q
T	T	T	F	F
T	F	F	T	T
F	T	T	F	F
F	F	T	F	F
			≡	

~(p → q) and p ∧ ~q have the same truth values, so they are logically equivalent.

(14b)

1) If n is prime and n is not odd, then n = 2.
2) If n is prime and n is not 2, then n is odd.

(17) (p ∧ q) → r ≡ (~p ∨ ~q) ∨ r

p	q	r	p ∧ q	(p ∨ q) → r	(~p ∨ ~q) ∨ r
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	F	T	T
F	T	F	F	T	T
F	F	T	F	T	T
F	F	F	F	T	T
				≡	

The statements have the same truth values, so they are logically equivalent.

(18)

p	q	r	p ∧ q → r	(~p ∨ ~q) ∨ r	~p ∧ ~q → ~r
T	T	T	T	T	T
T	T	F	T	T	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	F	T	T
F	F	F	T	T	T

Because they have the same truth values, the first two statements are logically equivalent.

(19)

~(p → q) ≡ p → ~q
~(~p ∨ q) ≡ ~p ∨ ~q
~~p ∧ ~q ≡ ~(p ∧ q)
p ∧ ~q ≡ ~(p ∧ q)

p	q	p ∧ ~q	~(p ∧ q)
T	T	F	F
T	F	T	T
F	T	F	T
F	F	F	T

False.

(20)

b) Today is NYE and tomorrow is not January.
c) The decimal expansion of r is terminating and r is not rational.
e) x is nonnegative, and x is not positive and x is not 0.
g) 6 | n, and 2 ∤ n or 3 ∤ n.

(22)

b) conv: If tomorrow is January, then today is NYE.
inv: If today is not NYE, then tomorrow is not January.
c) conv: If r ∈ ℚ, then r's decimal representation terminates.
inv: If the decimal representation of r does not terminate, then r is not rational.
e) conv: If x > 0 or x = 0, then x is nonnegative.
inv: If x is not gt 0 and x ≠ 0, then x is negative.
g) conv: If 2 | n and 3 | n, then 6 | n.
inv: If 6 ∤ n, then 2 ∤ n or 3 ∤ n.

(25)

p	q	p → q		p ∨ ~q
T	T	T		T
T	F	F	≠	T
F	T	T		T
F	F	T	≠	F
			≠	

(30) $p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$

p	q	r	$q \vee r$	$p \wedge q$	$p \wedge r$
T	T	T	T	T	T
T	T	F	T	T	F
T	F	T	T	F	T
T	F	F	F	F	F
F	T	T	T	F	F
F	T	F	T	F	F
F	F	T	T	F	F
F	F	F	F	F	F

$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T
T	T
T	T
F	F
F	F
F	F
F	F
F	F

$\equiv$

tautology

(32) If this quadratic equation has 2 real roots then its discriminant is > 0 and if the discriminant is > 0 then this quadratic equation has 2 real roots.

(33) If this integer is even then it is equal to twice some integer and if this integer is equal to twice some integer then it is even.

(41) If this triangle has two 45° angles, then it is a right triangle.

(42) $\sim p \rightarrow \sim q$

If 3 $\nmid n$, then 9 $\nmid n$.

$q \rightarrow p$

If $9 \mid n$, then $3 \mid n$.

(45) If the computer program produces errors during translation, then it is not correct.

(50) $(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \rightarrow r)$

$(p \rightarrow (\sim q \vee r)) \leftrightarrow (\sim(p \wedge q) \vee r)$

$(\sim p \vee (\sim q \vee r)) \leftrightarrow (\sim(p \wedge q) \vee r)$

(a) $\underbrace{(\sim(\sim p \vee (\sim q \vee r)) \vee (\sim(p \wedge q) \vee r))} \wedge \underbrace{(\sim(\sim(p \wedge q) \vee r) \vee (\sim p \vee (\sim q \vee r)))}$

$(\sim\sim p \wedge \sim(\sim q \vee r)) \vee \sim((p \wedge q) \wedge \sim r)$

$(p \wedge (q \wedge \sim r)) \vee \sim((p \wedge q) \wedge \sim r)$

$\sim(\sim(p \wedge (q \wedge \sim r)) \wedge ((p \wedge q) \wedge \sim r))$

$((p \wedge q) \wedge \sim r) \vee \sim(p \wedge \sim(\sim q \vee r))$

$((p \wedge q) \wedge \sim r) \vee \sim(p \wedge (q \wedge \sim r))$

$\sim(\sim((p \wedge q) \wedge \sim r) \wedge (p \wedge (q \wedge \sim r)))$

(b) $\left(\sim(\sim(p \wedge (q \wedge \sim r)) \wedge ((p \wedge q) \wedge \sim r))\right) \wedge \left(\sim(\sim((p \wedge q) \wedge \sim r) \wedge (p \wedge (q \wedge \sim r)))\right)$