

2.3 Valid and Invalid Arguments

p.61 #6-11, 22-32, 36-40, 42-44

⑥

premises		conclusion	
P	q	$P \rightarrow q$	$q \rightarrow r$
T	T	T	T
T	F	F	—
F	T	T	F
F	F	T	T

$P \vee q$

(F)

The argument is invalid. In row 4, the premises are true but the conclusion is false.

⑦

P	q	r	$P \rightarrow q$	$\sim q \vee r$	r
T	T	T	T	T	T
T	T	F	T	F	—
T	F	T	F	—	—
T	F	F	F	—	—
F	—	—	—	—	—
F	—	—	—	—	—
F	—	—	—	—	—

The argument is valid.

⑧

P	q	r	$P \vee q$	$P \rightarrow \sim q$	$P \rightarrow r$	r
T	T	T	T	F	—	—
T	T	F	T	F	—	—
T	F	T	T	T	T	T
T	F	F	T	T	F	—
F	T	T	T	T	T	T
F	F	T	F	T	T	(F)
F	F	F	F	—	—	—

The argument is invalid. See row 6.

⑨

P	q	r	$P \wedge q \rightarrow \sim r$	$P \vee \sim q \rightarrow P$	$\sim r$
T	T	T	F	—	T
T	T	F	T	T	(F)
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	T	F	—
F	F	T	T	F	—
F	F	F	T	F	—

The argument is invalid. See row 3.

⑩

P	q	r	$P \rightarrow q$	$q \rightarrow r$	$P \vee q \rightarrow r$
T	T	T	T	T	T
T	T	F	F	F	—
T	F	T	F	—	—
T	F	F	F	—	—
F	T	T	T	T	T
F	T	F	T	F	—
F	F	T	T	T	T
F	F	F	T	T	T

The argument is valid.

⑪

P	q	r	$P \rightarrow q \vee r$	$\sim q \vee \sim r$	$\sim P \vee \sim r$
T	T	T	T	F	—
T	T	F	T	T	(F)
T	F	T	T	T	—
T	F	F	F	—	—
F	T	T	T	F	—
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

The argument is invalid. See row 3.

⑫ $\sim A \rightarrow B$
 $\sim B \rightarrow A$

$\therefore \sim A \vee \sim B$

A	B	$\sim A \rightarrow B$	$\sim B \rightarrow A$	$\sim A \vee \sim B$
T	T	T	T	(F)
T	F	T	T	T
F	T	T	T	T
F	F	F	—	—

The argument is invalid. See row 1.

⑬ $j \vee e$
 $j \rightarrow m$
 $\therefore e \vee \sim m$

j	e	m	$j \vee e$	$j \rightarrow m$	$e \vee \sim m$
T	T	T	T	T	T
T	T	F	T	F	(F)
T	F	T	T	T	—
T	F	F	T	F	—
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	F	—	—
F	F	F	F	—	—

The argument is invalid. See row 3.

(24) $c \rightarrow t$
 t Invalid. Converse error.
 $\therefore c$

(25) $r \vee i$
 $\sim r$ Valid. Elimination.
 $\therefore i$

(26) $m \rightarrow \sim h$
 $\sim h \rightarrow \sim e$ Valid. Transitivity.
 $\therefore m \rightarrow \sim e$

(27) $t \rightarrow f$
 $\sim t$ Invalid. Inverse error.
 $\therefore \sim f$

(28) $e \rightarrow i$
 i Invalid. Converse error.
 $\therefore e$

(29) $d \rightarrow p$
 $\sim d$ Invalid. Inverse error.
 $\therefore \sim p$

(30) $c \rightarrow o$
 o Invalid. Converse error.
 $\therefore c$

(31) $j \wedge c$
 $\therefore c$ Valid. Specialization.

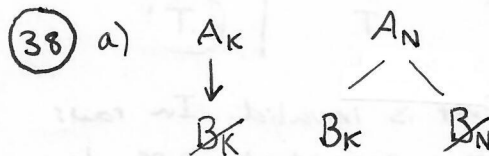
(32) $b \rightarrow s$
 $m \rightarrow s$ Valid. Modus Ponens.
 $\therefore b \vee m \rightarrow s$

(36) $u \vee x$
 $x \rightarrow s \vee m$
 $\sim s$
 $\sim m$
 $\therefore u$

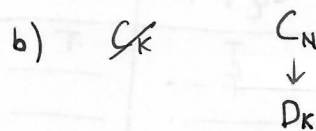
There is an undeclared variable.

(37) $L \rightarrow \sim K$
 $E \rightarrow K$
 L
 $E \vee F$
 $O \rightarrow G$
 $\therefore F$

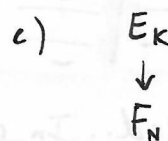
The treasure is buried under the flagpole.



A: Knave
B: Knight



C: Knave
D: Knight



E: Knight
F: Knave

d) U: If knight, contradiction.
 $\hookrightarrow U: \text{Knave}$
 $Z_K \rightarrow W_N$
 $\hookrightarrow Z: \text{Knave}$

$Y_K - X_N - W_K - V_N - \checkmark$

K: {W, Y}

N: {U, V, X, Z}

- 39 K
 LVS
 $C \rightarrow B$
 $L \rightarrow F$
 $\sim C \rightarrow \sim S$
 $S \rightarrow W$
 $\therefore F \wedge K$

The chauffeur killed Lord Hazelton.

Truth	Notes	Killer
A	$\sim B \therefore$ Muscles Killer Contradiction	X
B	$\sim A \therefore$ Lefty not killer $\sim C \therefore$ Muscles/Socko suspect $\sim D \therefore$ Lefty killer	X
C	not Muscles/Socko $\sim A \therefore$ Lefty not killer $\sim D \therefore$ Lefty killer	X
D	Lefty innocent $\sim A \therefore$ Lefty innocent $\sim B \therefore$ Muscles killer $\sim C \therefore$ Muscles/Socko suspicious	Muscles

Muscles killed Sharky.

- 42
- | | |
|--|-------------------------|
| (1) $q \rightarrow r$
$\sim r$
$\therefore \sim q$ | b
d
Modus Tollens |
| (2) $\sim q \rightarrow u \wedge s$
$\sim q$
$\therefore u \wedge s$ | e
1
Modus Ponens |
| (3) $p \vee q$
$\sim q$
$\therefore p$ | a
1
Elimination |
| (4) $u \wedge s$
$\therefore s$ | 2
Specialization |
| (5) p
s
$\therefore p \wedge s$ | 3
4
Conjunction |
| (6) $p \wedge s \rightarrow t$
$p \wedge s$
$\therefore t$ | c
5
Modus Ponens |

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- | | |
|--|-------------------------|
| (1) $u \vee w$
$\sim w$
$\therefore u$ | e
d
Elimination |
| (2) $u \rightarrow \sim p$
u
$\therefore \sim p$ | e
1
Modus Ponens |
| (3) $\sim p \rightarrow r \wedge \sim s$
$\sim p$
$\therefore r \wedge \sim s$ | a
2
Modus Ponens |
| (4) $r \wedge \sim s$
$\therefore \sim s$ | 3
Specialization |
| (5) $t \rightarrow s$
$\sim s$
$\therefore \sim t$ | b
4
Modus Tollens |

- 44
- | | |
|--|-------------------------|
| (1) $\sim s \rightarrow \sim t$
$\sim s$
$\therefore \sim t$ | c
e
Modus Ponens |
| (2) $r \vee s$
$\sim s$
$\therefore r$ | b
e
Elimination |
| (3) $\sim q \vee s$
$\sim s$
$\therefore \sim q$ | d
e
Elimination |
| (4) $p \rightarrow q$
$\sim q$
$\therefore \sim p$ | a
3
Modus Tollens |
| (5) $\sim p$
r
$\therefore \sim p \wedge r$ | 4
2
Conjunction |
| (6) $\sim p \wedge r \rightarrow u$
$\sim p \wedge r$
$\therefore u$ | f
5
Modus Ponens |
| (7) $w \vee t$
$\sim t$
$\therefore w$ | g
1
Elimination |
| (8) u
w
$u \wedge w$ | 6
7
Conjunction |