

# 3.1 Predicates + Quantified Statements I

p. 106 #1, 4-8, 10-12, 14, 16, 18-19, 22-23, 28-30

- ① a) F    b) T    c) F  
 d) T    e) F    f) T

- ④ Q(2):  $4 \leq 30$  True  
 Q(-2):  $4 \leq 30$  True  
 a) Q(7):  $49 \leq 30$  False  
 Q(-7):  $49 \leq 30$  False

- b)  $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$   
 c)  $\{1, 2, 3, 4, 5\}$

- ⑤ a) Q(-2, 1)  $-2 < 1$  ✓  
                   ↓  
                    $4 < 1$  X

b)  $x = -1$      $y = 0$

- c) Q(3, 8)  $3 < 8$   
                   ↓  
                    $9 < 64$  ✓

a)  $x = 0$      $y = 1$

- ⑥ a) R(25, 10)  $25 \mid 10^2$  ✓  
                   ↓  
                    $25 \mid 10$  X

b)  $x = 8$      $y = 4$

- c) R(5, 10)  $5 \mid 10^2$  ✓  
                   ↓  
                    $5 \mid 10$  ✓

d)  $x = 2$      $y = 4$

- ⑦ a)  $\{-6, -3, -2, -1, 1, 2, 3, 6\}$   
 b)  $\{1, 2, 3, 6\}$   
 c)  $\{x \in \mathbb{R} \mid 1 \leq x \leq 2, -1 \gg x \gg -2\}$   
 d)  $\{-2, -1, 1, 2\}$

- ⑧ a)  $\{-9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$   
 b)  $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$   
 c)  $\{-8, -6, -4, -2, 0, 2, 4, 6, 8\}$

⑩  $a = 1$     ⑪  $m = 0$      $n = 1$

⑫  $x = 1$      $y = 1$

⑭  $b, c, e, f$

- ⑯ a)  $\forall x \in \text{dinosaurs}, x$  is extinct.  
 b)  $\forall x \in \mathbb{R}, x > 0$  or  $x < 0$  or  $x = 0$   
 c)  $\forall x \in \mathbb{R} \setminus \mathbb{Q}, x \in \mathbb{Z}$   
 d)  $\forall x \in \text{logicians}, x$  is not lazy.  
 e)  $\forall x \in \mathbb{Z}, x^2 \neq 2, 147, 581, 953$   
 f)  $\forall x \in \mathbb{R}, x^2 \neq -1$

- ⑰ a)  $\exists x \in D$  such that  $M(x)$  and  $E(x)$   
 b)  $\forall x \in D, C(x) \rightarrow E(x)$   
 c)  $\forall x \in D, C(x) \rightarrow \sim E(x)$   
 d)  $\exists x \in D$  such that  $C(x)$  and  $M(x)$   
 e)  $(\exists x \in D \text{ such that } C(x) \wedge E(x)) \wedge$   
        $(\exists x \in D \text{ such that } C(x) \wedge \sim E(x))$

⑱  $b, d, e$

22 a)  $\forall$  programs  $x$ , if  $x$  is a Java program, then  $x$  has at least 5 lines.

b)  $\forall$  arguments  $x$ , if  $x$  is valid and has true premises, then  $x$  has a true conclusion.

23 a) i.  $\forall x$ , if  $x$  is an equilateral triangle, then  $x$  is isosceles.

ii.  $\forall$  equilateral triangles  $x$ ,  $x$  is isosceles.

b) i.  $\forall x$ , if  $x$  is a computer science student, then  $x$  needs to take data structures.

ii.  $\forall$  computer science students  $x$ ,  $x$  needs to take data structures.

28 a) 0 is a positive real number.  
False.

b) If a real number is negative, its opposite is positive.  
True.

c) If a number is an integer, then it is real.  
True.

d) There exists a real number that is not an integer.  
True.

29 a) There exists a rectangle that is also a square.  
True.

b) There exists a rectangle that is not a square.  
True.

c) If a shape is a square, then it is a rectangle.  
True.

30 a) There is an integer that is prime and not odd.  
True.

b) If an integer is prime, then it is not a perfect square.  
True.

c) There is an integer which is odd and is also a perfect square.  
True.