4.3 Direct Proof and Counterexample III: Divisibility

p. 177 #1-13, 15, 19-22, 32-33, 36

(1) Yes. $52 = 13(4)$

(2) Yes. $56 = 7(8)$

(3) Yes. $0 = 5(0)$

(4) $(3k+1)(3k+2)(3k+3)$
$= 3((k+1)(3k+1)(3k+2))$ Yes.

(5) $6m(2m+10) = 12m^2 + 60m$
$= 4(3m^2 + 15m)$ Yes.

(6) No.

(7) Yes. $66 = -3(-22)$

(8) $ba(a+b) = 3a(2(a+b))$ Yes.

(9) $2a \cdot 34b = 2 \cdot 2 \cdot 17 \cdot a \cdot b = 4(17ab)$ Yes.

(10) No.

(11) No.

(12) $n = 4k+1 \quad n^2 = 16k^2 + 8k + 1$
$n^2 - 1 = 16k^2 + 8k = 8(2k^2 + k)$ Yes.

(13) $n = 4k+3 \quad n^2 = 16k^2 + 24k + 9$
$n^2 - 1 = 16k^2 + 24k + 8 = 8(2k^2 + 3k + 1)$ Yes.

(15) $a|b \rightarrow b = ak, k \in \mathbb{Z}$
$a|c \rightarrow c = aj, j \in \mathbb{Z}$
$b + c = ak + aj = a(k+j)$ Yes.

(19) $a|b \rightarrow b = ak, k \in \mathbb{Z}$
$bc = (ak)c = a(kc)$ True.

(20) Let $n \in \mathbb{Z}$
$(3n)(3n+1)(3n+2)$
$= 3(n(3n+1)(3n+2))$ True.

(21) Let $n = 2k, k \in \mathbb{Z}$
$(2k)(2k+2) = 4k^2 + 4k$
$= 4(k^2 + k)$ True.

(22) $6|n$
$2 \cdot 3 | n \rightarrow \boxed{2|n} \wedge 3|n$ True

(32) Let $S = \{72, 21, 15, 36, 69, 81, 9, 27, 42, 63\}$
$\forall n \in S, 3|n$
$3 \nmid 100$
$\therefore$ no subset of S will sum to 100.

(33) Let $S = \{5, 10, 25, 5, 10, 25, \cdots\}$
$\forall n \in S, 5|n$
$5 \nmid 472$
$\therefore$ no subset of S will sum to 472.

(36) a) (3) 4 (5) (9) $\sum$ digits = 54

b) (3) (4) 5 9 $\sum$ digits = 42

c) 3 (4) 5 9 $\sum$ digits = 61

d) (3) (4) 5 (9) $\sum$ digits = 45