4.4 Direct Proof & Counter-Example IV: The Quotient-Remainder Theorem

p. 189 #1-11, 13-15, 19, 21-24, 35-36

① $70 = 7(9) + 7$ $d=7$ $r=7$

② $62 = 8(7) + 6$ $d=8$ $r=6$

③ $36 = 0(40) + 36$ $d=0$ $r=36$

④ $3 = 0(11) + 3$ $d=0$ $r=3$

⑤ $-45 = -5(11) + 10$ $d=-5$ $r=10$

⑥ $-27 = -4(8) + 5$ $d=-4$ $r=5$

⑦ a) $43 \text{ div } 9 = 4$

b) $43 \bmod 9 = 7$

⑧ a) $50 \text{ div } 7 = 7$

b) $50 \bmod 7 = 1$

⑨ a) $28 \text{ div } 5 = 5$

b) $28 \bmod 5 = 3$

⑩ a) $30 \text{ div } 2 = 15$

b) $30 \bmod 2 = 0$

⑪ a) $(6+15) \bmod 7 = 21 \bmod 7 = 0$

$0 \rightarrow$ Sunday ✓

b) $(0+7) \bmod 7 = 7 \bmod 7 = 0$

$0 \rightarrow$ Sunday ✓

c) $(4+12) \bmod 7 = 16 \bmod 7 = 2$

$2 \rightarrow$ Tuesday

⑬ Monday: 1

$(1+30) \bmod 7 = 31 \bmod 7 = 3$

$3 \rightarrow \boxed{\text{Wednesday}}$

⑭ Tuesday: 2

$(2+1000) \bmod 7 = 1002 \bmod 7 = 1$

$1 \rightarrow \boxed{\text{Monday}}$

⑮ $50 \cdot 365$ days/yr + 13 leap days

$(6+18263) \bmod 7 = 18269 \bmod 7 = 6$

$6 \rightarrow \boxed{\text{Saturday}}$

⑲ $n^2 - n + 3 = \underbrace{n(n-1)}_{\text{one of these is even}} + 2 + 1$

$n(n-1) + 2 + 1 \bmod 2 = 1$

㉑ $b \bmod 12 = 5$

$b = 12k + 5, \ k \in \mathbb{Z}$

$8b = 8(12k) + 40$

$8b = 8(12k) + 3(12) + 4$

$8b \bmod 12 = \boxed{4}$

㉒ $c \bmod 15 = 3$

$c = 15k + 3, \ k \in \mathbb{Z}$

$10c = 10(15k) + 30$

$10c = 15(10k + 2)$

$10c \bmod 15 = \boxed{0}$

(23) $n \bmod 5 = 3$

$n = 5k+3,\ k \in \mathbb{Z}$

$n^2 = (5k+3)^2 = 25k + 30k + 9$

$n^2 = 5(11k+1) + 4$

$n^2 \bmod 5 = \boxed{4}$

(24) The problem statement is flawed.

(35) Even: $n = 2k,\ k \in \mathbb{Z}$

$$n^4 = (2k)^4 = 16k^4 = 8(2k^4) \checkmark$$

Odd: $m = 2j+1,\ j \in \mathbb{Z}$

$$m^4 = (2j+1)^4 = 16j^4 + 32j^3 + 24j^2 + 8j + 1$$

$$= 8(2j^4 + 4j^3 + 3j^2 + j) + 1 \checkmark$$

(36) Let $n = 4q,\ q \in \mathbb{Z}$

$$\underbrace{(4q)(4q+1)(4q+2)(4q+3)}_{\text{4 consecutive integers}} = 256q^4 + 384q^3 + 176q^2 + 24q$$

$$= 8(32q^4 + 48q^3 + 22q^2 + 3q) \bmod 8 = 0 \checkmark$$