

# 5.1 Sequences

p. 243 # 11-16 all, 19-32 all, 41-75 odd

$$(11) a_n = n(-1)^{n-1} \text{ for } n \geq 0$$

$$(12) a_n = \frac{n}{(n+1)^2} \text{ for } n \geq 1$$

$$(13) a_n = \frac{1}{n} - \frac{1}{n+1} \text{ for } n \geq 1$$

$$(14) a_n = \frac{n^2}{3^n} \text{ for } n \geq 1$$

$$(15) a_n = \left(1 - \frac{1}{n}\right)(-1)^{n-1} \text{ for } n \geq 1$$

$$(16) a_n = 3 \cdot 2^n \text{ for } n \geq 0$$

$$(19) \sum_{k=1}^5 (k+1) = 2+3+4+5+6 = \boxed{20}$$

$$(20) \prod_{k=2}^4 k^2 = 4 \cdot 9 \cdot 16 = \boxed{576}$$

$$(21) \sum_{m=0}^3 \frac{1}{2^m} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \boxed{\frac{15}{8}}$$

$$(22) \prod_{j=0}^4 (-1)^j = (1)(-1)(1)(-1)(1) = \boxed{1}$$

$$(23) \sum_{i=1}^1 i(i+1) = 1(1+1) = \boxed{2}$$

$$(24) \sum_{j=0}^0 (j+1) \cdot 2^j = (0+1) \cdot 2^0 = \boxed{1}$$

$$(25) \prod_{k=2}^2 \left(1 - \frac{1}{k}\right) = \left(1 - \frac{1}{2}\right) = \boxed{\frac{1}{2}}$$

$$(26) \sum_{k=-1}^1 (k^2+3) = 4+3+4 = \boxed{11}$$

$$(27) \sum_{n=1}^{10} \left(\frac{1}{n} - \frac{1}{n+1}\right) = 1 - \frac{1}{11} = \boxed{\frac{10}{11}}$$

$$\left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \dots + \left(\frac{1}{10} - \frac{1}{11}\right)$$

$$(28) \prod_{i=2}^5 \frac{i(i+2)}{(i-1)(i+1)} = \frac{2 \cdot 4}{1 \cdot 3} \cdot \frac{3 \cdot 5}{2 \cdot 4} \cdot \frac{4 \cdot 6}{3 \cdot 5} \cdot \frac{5 \cdot 7}{4 \cdot 6} = \boxed{\frac{35}{3}}$$

$$(29) \sum_{i=1}^n (-2)^i = -2^1 + 2^2 - 2^3 + \dots + (-2)^n$$

$$(30) \sum_{j=1}^n j(j+1) = 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \dots + n \cdot (n+1)$$

$$(31) \sum_{k=0}^{n+1} \frac{1}{k!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{(n+1)!}$$

$$(32) \sum_{i=1}^{k+1} i(i!) = 1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + (k+1)(k+1)!$$

$$(41) \sum_{k=1}^m \frac{k}{k+1} + \frac{m+1}{m+2} = \sum_{k=1}^{m+1} \frac{k}{k+1}$$

$$(43) \sum_{i=1}^7 i^2 (-1)^{i+1}$$

$$(45) \prod_{i=2}^4 (i^2 - 1)$$

$$(47) \sum_{i=0}^5 (-r)^i$$

$$(49) \sum_{k=1}^n k^3$$

$$(51) \sum_{k=0}^{n-1} (n-k)$$

$$(53) \quad i = k+1 \quad k = i-1$$

$$\sum_{k=0}^5 k(k-1) \Rightarrow \sum_{i=1}^6 (i-1)(i-2)$$

$$(55) \quad j = i-1 \quad i = j+1$$

$$\sum_{i=1}^{n+1} \frac{(i-1)^2}{i \cdot n} \Rightarrow \sum_{j=0}^n \frac{j^2}{n(j+1)}$$

$$(57) \quad j = i-1 \quad i = j+1$$

$$\sum_{i=1}^{n-1} \frac{i}{(n-1)^2} \Rightarrow \sum_{j=0}^{n-2} \frac{j+1}{(n-j-1)^2}$$

$$(59) \quad \sum_{k=1}^n (2k-3) + \sum_{k=1}^n (4-5k) = \sum_{k=1}^n (1-3k)$$

$$(61) \quad \left( \prod_{k=1}^n \frac{k}{k+1} \right) \left( \prod_{k=1}^n \frac{k+1}{k+2} \right) = \prod_{k=1}^n \frac{k}{k+2}$$

$$(63) \quad \frac{6!}{8!} = \frac{6!}{8 \cdot 7 \cdot 6!} = \boxed{\frac{1}{56}}$$

$$(65) \quad \frac{n!}{(n-1)!} = \frac{n \cdot (n-1)!}{(n-1)!} = \boxed{n}$$

$$(67) \quad \frac{n!}{(n-2)!} = \frac{n \cdot (n-1) \cdot (n-2)!}{(n-2)!} = \boxed{n^2 - n}$$

$$(69) \quad \frac{n!}{(n-k)!} = \frac{n(n-1)(n-2) \dots (n-k+1) \cdot \cancel{(n-k)!}}{\cancel{(n-k)!}}$$

$$= n(n-1)(n-2) \dots (n-k+1)$$

$$= \prod_{i=0}^{k-1} (n-i)$$

$$(71) \quad \binom{5}{3} = \frac{5!}{3!2!} = \frac{5 \cdot 4 \cdot 3!}{3! \cdot 2} = \boxed{10}$$

$$(73) \quad \binom{3}{0} = \frac{3!}{0!3!} = \frac{1}{1} = \boxed{1}$$

$$(75) \quad \binom{n}{n-1} = \frac{n!}{(n-1)!(1!)} = \frac{n \cdot \cancel{(n-1)!}}{\cancel{(n-1)!}} = \boxed{n}$$