

5.2 Mathematical Induction I

p. 257 #7-9, 11-14, 19

⑦ $\forall n \in \mathbb{Z} | n \geq 1, 1+6+11+\dots+(5n-4) = \frac{n(5n-3)}{2}$

$P(1) = 1 = \frac{(1)(5(1)-3)}{2} = \frac{1 \cdot 2}{2} = 1 \checkmark$

$P(k+1) = 1+6+11+\dots+(5k-4) + 5(k+1)-4$

$\frac{k(5k-3)}{2} + 5k+5-4 = \frac{(k+1)(5(k+1)-3)}{2}$

$k(5k-3) + 10k+2 = (k+1)(5k+2)$

$5k^2 + 7k + 2 = 5k^2 + 7k + 2 \checkmark$

True by M.I.

⑧ $\forall n \in \mathbb{Z} | n \geq 0, 1+2^1+2^2+\dots+2^n = 2^{n+1} - 1$

$P(0) = 1 = 2^1 - 1 = 1 \checkmark$

$P(k+1) = 1+2^1+2^2+\dots+2^k + 2^{k+1} = 2^{k+2} - 1$

$(2^{k+1} - 1) + 2^{k+1} = 2^{k+2} - 1$

$2^1(2^{k+1}) - 1 = 2^{k+2} - 1$

$2^{k+2} - 1 = 2^{k+2} - 1 \checkmark$

True by M.I.

⑨ $\forall n \in \mathbb{Z} | n \geq 3, 4^3+4^4+4^5+\dots+4^n = \frac{4(4^n-16)}{3}$

$P(3) = 4^3 = \frac{4(4^3-16)}{3} = \frac{4 \cdot 48}{3} = 64 \checkmark$

$P(k+1) = 4^3+4^4+4^5+\dots+4^k + 4^{k+1} = \frac{4(4^{k+1}-16)}{3}$

$\frac{4(4^k-16)}{3} + 4^{k+1} = \frac{4(4^{k+1}-16)}{3}$

$4(4^k-16) + 3 \cdot 4^{k+1} = 4(4^{k+1}-16)$

$4^k - 16 + 3 \cdot 4^k = 4^{k+1} - 16$

$4 \cdot 4^k = 4^{k+1}$

$4^{k+1} = 4^{k+1} \checkmark$

True by M.I.

⑪ $\forall n \in \mathbb{Z} | n \geq 1, 1^3+2^3+\dots+n^3 = \left[\frac{n(n+1)}{2} \right]^2$

$P(1) = 1^3 = 1 = \left[\frac{1(1+1)}{2} \right]^2 = \left(\frac{2}{2} \right)^2 = 1 \checkmark$

$P(k+1) = 1^3+2^3+\dots+k^3 + (k+1)^3 = \left[\frac{(k+1)(k+2)}{2} \right]^2$

$\left[\frac{k(k+1)}{2} \right]^2 + (k+1)^3 = \frac{(k+1)^2(k+2)^2}{4}$

$(k+1)^2 \left[\frac{k^2}{4} + (k+1) \right] = (k+1)^2 \frac{(k+2)^2}{4}$

$k^2 + 4k + 4 = (k+2)^2$

$k^2 + 4k + 4 = k^2 + 4k + 4 \checkmark$

True by M.I.

⑫ $\forall n \in \mathbb{Z} | n \geq 1, \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

$P(1) = \frac{1}{2} = \frac{1}{1+1} = \frac{1}{2} \checkmark$

$P(k+1) = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2}$

$\frac{k}{k+1} + \frac{1}{(k+1)(k+2)} = \frac{(k+2)k}{(k+1)(k+2)} + \frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2}$

$\frac{k^2+2k+1}{(k+1)(k+2)} \rightarrow \frac{(k+1)(k+1)}{(k+1)(k+2)} = \frac{k+1}{k+2} \checkmark$

True by M.I.

⑬ $\forall n \in \mathbb{Z} | n \geq 2, \sum_{i=1}^{n-1} i(i+1) = \frac{n(n-1)(n+1)}{3}$

$P(2) = \sum_{i=1}^{2-1} i(i+1) = 1(1+1) = 2 = \frac{2(2-1)(2+1)}{3} = \frac{2 \cdot 1 \cdot 3}{3} = 2 \checkmark$

$P(k+1) = \sum_{i=1}^{k-1} i(i+1) + k(k+1) = \frac{(k+1)(k)(k+2)}{3}$

$\frac{k(k-1)(k+1)}{3} + k(k+1) = \frac{(k+1)(k)(k+2)}{3}$

$k(k-1)(k+1) + 3k(k+1) = k(k+1)(k+2)$

$k(k-1) + 3k = k(k+2)$

$k^2 - k + 3k = k^2 + 2k$

$k^2 + 2k = k^2 + 2k$

True by M.I.

