

5.6 Defining Sequences Recursively

p.302 #3,5,7,10-11,14-15,22-27,38,40

③ $C_0 = 1$
 $C_1 = 1(1)^2 = 1$
 $C_2 = 2(1)^2 = 2$
 $C_3 = 3(2)^2 = 12$
 $C_4 = 4(12)^2 = 576$

⑤ $S_0 = 1$
 $S_1 = 1$
 $S_2 = 1 + 2(1) = 3$
 $S_3 = 3 + 2(1) = 5$
 $S_4 = 5 + 2(3) = 11$

⑦ $u_1 = 1$
 $u_2 = 1$
 $u_3 = 3(1) - 1 = 2$
 $u_4 = 4(2) - 1 = 7$
 $u_5 = 5(7) - 2 = 33$

⑩ $\forall k \in \mathbb{Z} | k \geq 1, b_k = 4b_{k-1} \quad b_n = 4^n$
 $P(0): b_0 = 1 = 4^0 = 1 \checkmark$
 $P(n+1): b_{n+1} = 4b_n \quad b_n = 4^n$
 $b_{n+1} = 4 \cdot 4^n$
 $b_{n+1} = 4^{n+1} \checkmark$ True by M.I.

⑪ $\forall k \in \mathbb{Z} | k \geq 1, c_k = 2c_{k-1} + 1 \quad c_n = 2^n - 1$
 $P(0): c_0 = 2^0 - 1 = 0$
 $P(1): c_1 = 2^1 - 1 = 1 = 2c_0 + 1 = 0 + 1 = 1 \checkmark$
 $P(n+1): c_{n+1} = 2c_n + 1 \quad c_n = 2^n - 1$
 $c_{n+1} = 2(2^n - 1) + 1$
 $c_{n+1} = 2^{n+1} - 2 + 1$
 $c_{n+1} = 2^{n+1} - 1 \checkmark$ True by M.I.

⑭ $\forall k \in \mathbb{Z} | k \geq 2, d_k = 5d_{k-1} - 6d_{k-2} \quad d_n = 3^n - 2^n$
 $P(2): d_2 = 5d_1 - 6d_0 \quad d_1 = 3^1 - 2^1 = 1$
 $d_2 = 5 - 0 = 5 \quad d_0 = 3^0 - 2^0 = 0$
 $d_2 = 3^2 - 2^2 = 5 \checkmark$
 $P(n+1): d_{n+1} = 5d_n - 6d_{n-1} \quad d_n = 3^n - 2^n$
 $d_{n+1} = 5 \cdot 3^n - 5 \cdot 2^n - 6 \cdot 3^{n-1} + 6 \cdot 2^{n-1}$
 $d_{n+1} = 5 \cdot 3^n - 5 \cdot 2^n - 2 \cdot 3^n + 3 \cdot 2^n$
 $d_{n+1} = 3 \cdot 3^n - 2 \cdot 2^n = 3^{n+1} - 2^{n+1} \checkmark$
True by M.I.

⑮ $C_n = \frac{1}{n+1} \binom{2n}{n}$ by definition

$$C_n = \frac{(2n)!}{(n+1)n!n!}$$

Show: $\frac{1}{4n+2} \binom{2n+2}{n+1} = C_n$

$$\frac{1(2n+2)(2n+1)(2n)!}{(4n+2)(n+1)n!(n+1)n!}$$

$$\frac{2(n+1)(2n+1)(2n)!}{2(2n+1)(n+1)n!(n+1)n!} = \frac{(2n)!}{(n+1)n!n!} \checkmark$$

⑲ a) $r_n = r_{n-1} + 4r_{n-2} \quad \forall n \geq 1 \quad r_0 = 1, r_1 = 1$

b)

n	r _n
0	1
1	1
2	5
3	9
4	29
5	65
6	181

 c) 99322 rabbits (49661 pairs)

⑳ a) $S_n = S_{n-1} + 3S_{n-3} \quad \forall n \geq 1 \quad S_0 = 5, S_1 = 5$

b)

n	S _n
0	1
1	1
2	1
3	4
4	7
5	10

 c) 1808 rabbits (904 pairs)

㉑ 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377

㉒ a) Any term of F is the sum of the two previous terms. F_{k-1} and F_k are the terms preceding F_{k+1} , so the equation is valid.

b) $F_{k+2} = F_{k+1} + F_k$

c) $F_{k+3} = F_{k+2} + F_{k+1}$

