6.1 Set Theory – Definitions and the Element Method of Proof

p.349 #1, 5-6, 10-11, 13-14, 17, 20, 22, 25, 27, 30

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(1) a) $A \subset B$, $A \subseteq B$

b) $A \subseteq B$, $B \subseteq A$

c) N/A

d) N/A

e) $B \subset A$, $B \subseteq A$

f) $A \subseteq B$, $B \subseteq A$

(5) a) Suppose x is a particular but arbitrarily chosen element of C.

By def'n of C, $\exists r \in \mathbb{Z} \mid x = 6r - 5$

Let $s = 2r - 2$, $s \in \mathbb{Z}$.

$r = \frac{s+2}{2}$ $\quad x = 6\left(\frac{s+2}{2}\right) - 5 = 3s + 1$

$\therefore x \in D \quad \therefore C \subseteq D$

b) $D \not\subseteq C$. Counterexample:

$4 \in D$, as $3s+1 = 4$ when $s = 1$.

However, $\forall r \in \mathbb{Z}$, $6r - 5 \neq 4$.

(6) a) $A \not\subseteq B$. Counterexample:

$12 \in A$, as $5a + 2 = 12$ when $a = 2$.

However, $\forall b \in \mathbb{Z}$, $10b - 3 \neq 12$.

b) $x \in B$ (yada yada)

By def'n of B, $\exists b \in \mathbb{Z} \mid 10b - 3 = x$

Let $a = 2b - 1$, $a \in \mathbb{Z}$

$b = \frac{a+1}{2}$ $\quad x = 10\left(\frac{a+1}{2}\right) - 3 = 5a + 2$

$\therefore x \in A \quad \therefore B \subseteq A$

c) $x \in B$

$x = 10b - 3, b \in \mathbb{Z}$

Let $b = c + 1, c \in \mathbb{Z}$

$x = 10(c+1) - 3$

$x = 10c + 7$

$\therefore x \in C$

$\therefore B \subseteq C$

$x \in C$

$x = 10c + 7$, $c \in \mathbb{Z}$

Let $c = b - 1, b \in \mathbb{Z}$

$x = 10(b-1) + 7$

$x = 10b - 3$

$\therefore x \in B$

$\therefore C \subseteq B$

$\boxed{B = C}$

(10) a) $\{1,3,5,6,7,9\}$ b) $\{3, 9\}$

c) $\{1,2,3,4,5,6,7,8,9\}$ d) $\{\}$

e) $\{1, 5, 7\}$ f) $\{6\}$

g) $\{2,3,4,6,8,9\}$ h) $\{6\}$

(11) a) $(0,4)$ b) $[1,2]$ c) $(-\infty,0] \cup (2,\infty)$

d) $(0,2] \cup [3,9)$ e) $\{\}$ f) $(-\infty,1) \cup [4,\infty)$

g) $(-\infty,0] \cup [4,\infty)$ h) $(-\infty,1) \cup (2,\infty)$

i) $(-\infty,1) \cup (2,\infty)$ j) $(-\infty,0] \cup [4,\infty)$

(13) a) True

b) True

c) False

d) False

e) True

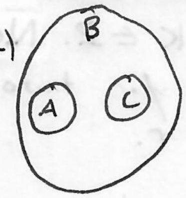
f) True

g) True

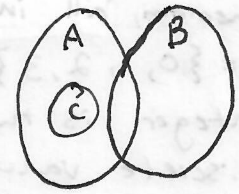
h) True

i) False

(14) a)

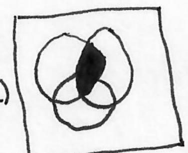


b)



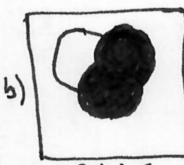
(17)

a)



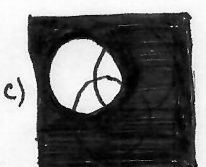
$A \cap B$

b)



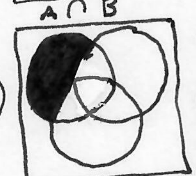
$B \cup C$

c)



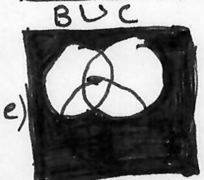
A^c

d)



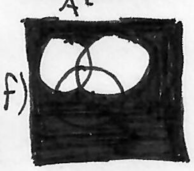
$A - (B \cup C)$

e)



$(A \cup B)^c$

f)



$A^c \cap B^c$

(20) a) $[0, 4]$ b) $[0, 1]$

c) No. $B_1 \cap B_2 = [0, 1] \neq \emptyset$

(22) a) $\bigcup_{i=0}^{4} D_i = [-4,4]$ b) $\bigcap_{i=0}^{4} D_i = \{0\}$

c) No. $\forall i \in \mathbb{Z} \mid i \geq 0,\ 0 \in D_i$.

d) $\bigcup_{i=0}^{n} D_i = [-n, n]$ e) $\bigcap_{i=0}^{n} D_i = \{0\}$

f) $\bigcup_{i=0}^{\infty} D_i = \mathbb{R}$ g) $\bigcap_{i=0}^{\infty} D_i = \{0\}$

(25) a) $\bigcup_{i=1}^{4} R_i = [1,2]$ b) $\bigcap_{i=1}^{4} R_i = [1, \frac{5}{4}]$

c) No. $\forall i \in \mathbb{Z} \mid i \geq 1,\ 1 \in R_i$.

d) $\bigcup_{i=1}^{n} R_i = [1,2]$ e) $\bigcap_{i=1}^{n} R_i = [1, \frac{n+1}{n}]$

f) $\bigcup_{i=1}^{\infty} R_i = [1,2]$ g) $\bigcap_{i=1}^{\infty} R_i = \{1\}$

(27) a) No. b) Yes. c) No.

d) No. e) Yes.

(30) Yes. By the quotient remainder theorem, $\forall x \in \mathbb{Z}$, $x = 4K + r$ where $r = \{0, 1, 2, 3\}$ and $K \in \mathbb{Z}$. For no integer is this true for two discrete values of r.