

6.1 Set Theory - Definitions and the Element Method of Proof

p.349 #1, 5-6, 10-11, 13-14, 17, 20, 22, 25, 27, 30

- ① a) $A \subset B, A \subseteq B$
 b) $A \subseteq B, B \subseteq A$
 c) N/A
 d) N/A
 e) $B \subset A, B \subseteq A$
 f) $A \subseteq B, B \subseteq A$

⑤ a) Suppose x is a particular but arbitrarily chosen element of C .

By def'n of $C, \exists r \in \mathbb{Z} \mid x = 6r - 5$

Let $s = 2r - 2, s \in \mathbb{Z}$.

$r = \frac{s+2}{2} \quad x = 6\left(\frac{s+2}{2}\right) - 5 = 3s + 1$

$\therefore x \in D \quad \therefore C \subseteq D$

b) $D \not\subseteq C$. Counterexample:

$4 \in D$, as $3s + 1 = 4$ when $s = 1$.

However, $\forall r \in \mathbb{Z}, 6r - 5 \neq 4$.

⑥ a) $A \not\subseteq B$. Counterexample:

$12 \in A$, as $5a + 2 = 12$ when $a = 2$.

However, $\forall b \in \mathbb{Z}, 10b - 3 \neq 12$.

b) $x \in B$ (yada yada)

By def'n of $B, \exists b \in \mathbb{Z} \mid 10b - 3 = x$

Let $a = 2b - 1, a \in \mathbb{Z}$

$b = \frac{a+1}{2} \quad x = 10\left(\frac{a+1}{2}\right) - 3 = 5a + 2$

$\therefore x \in A \quad \therefore B \subseteq A$

c) $x \in B$

$x = 10b - 3, b \in \mathbb{Z}$

Let $b = c + 1, c \in \mathbb{Z}$

$x = 10(c+1) - 3$

$x = 10c + 7$

$\therefore x \in C$

$\therefore B \subseteq C$

$x \in C$

$x = 10c + 7, c \in \mathbb{Z}$

Let $c = b - 1, b \in \mathbb{Z}$

$x = 10(b-1) + 7$

$x = 10b - 3$

$\therefore x \in B$

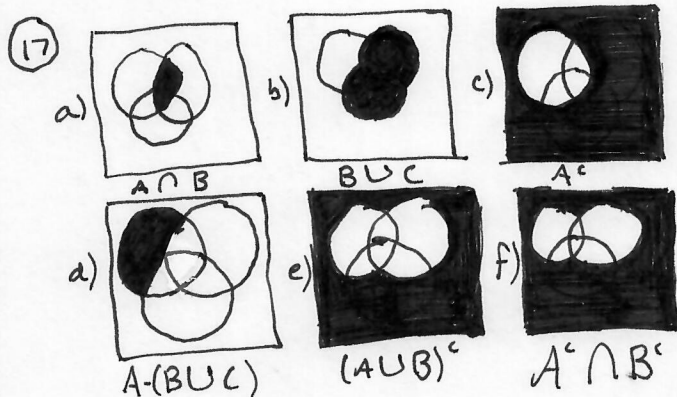
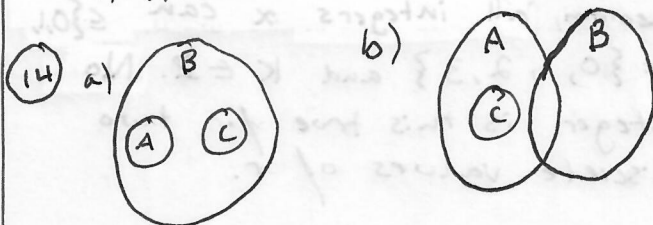
$\therefore C \subseteq B$

$B = C$

- ⑩ a) $\{1, 3, 5, 6, 7, 9\}$ b) $\{3, 9\}$
 c) $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ d) $\{\}$
 e) $\{1, 5, 7\}$ f) $\{6\}$
 g) $\{2, 3, 4, 6, 8, 9\}$ h) $\{6\}$

- ⑪ a) $(0, 4)$ b) $[1, 2]$ c) $(-\infty, 0] \cup (2, \infty)$
 d) $(0, 2] \cup [3, 9)$ e) $\{\}$ f) $(-\infty, 1) \cup [4, \infty)$
 g) $(-\infty, 0] \cup [4, \infty)$ h) $(-\infty, 1) \cup (2, \infty)$
 i) $(-\infty, 1) \cup (2, \infty)$ j) $(-\infty, 0] \cup [4, \infty)$

- ⑬ a) True f) True
 b) True g) True
 c) False h) True
 d) False i) False
 e) True



- ⑳ a) $[0, 4]$ b) $[0, 1]$
 c) No. $B_1 \cap B_2 = [0, 1] \neq \emptyset$

