5.1 – Eigenvalues and Eigenvectors

- Use this to instantly get the characteristic polynomial of a 2x2 matrix A:

$$0 = \lambda^2 - \mathrm{trace}(A) + \det(A) \qquad A = \begin{bmatrix} 4 & -3 \\ 7 & 2 \end{bmatrix} \quad \underbrace{0 = \lambda^2 - 6\lambda + 29}_{\text{char. pol.}}$$

- To check your eigenvalues using a calculator, set up this equation:
  Y = det(X*identity(n) - [A])
  where n is the matrix size and [A] is the matrix for which you seek eigenvalues. The eigenvalues will be the zeroes of the graph. You will also be able to visually identify multiplicity.
  det and identity are available under Matrix → Math on a TI 83-84

5.2 – Diagonalization

- It's easy to make mistakes along the way to calculating P. Once you've found it, check it immediately by verifying that $P^{-1}AP = D$, and that the positions of the eigenvalues in D correspond to the order of the eigenvectors in P. If they do not, go back and correct your error before moving on.
- $A^k = PD^kP^{-1}$

5.3 – Complex Vector Spaces

- Once you substitute one of your eigenvalues into $\lambda I - A$, don't try and RREF. Look at one of the rows, and evaluate it like so:

If you plug in λ and your matrix becomes this: $\begin{bmatrix} i-1 & 1 \\ \boxed{-2 \quad i+1} \end{bmatrix}$

$(-2)x_1 + (i+1)x_2 = 0$
let $x_2 = t$
then $-2x_1 = -(i+1)t$
$x_1 = \frac{-i-1}{-2}t = \frac{i+1}{2}t$

$\rightarrow t\boxed{\begin{bmatrix} \frac{i+1}{2} \\ 1 \end{bmatrix}}$ eigenvector

- For 2x2s: Once you calculate one of your eigenvectors, the second eigenvector is the conjugate of the first:

if $\begin{bmatrix} 1+i \\ 1 \end{bmatrix}$ is an eigenvector, $\begin{bmatrix} 1-i \\ 1 \end{bmatrix}$ is the other.

How to do the 5.4 problems

Use this guide alongside the posted solutions for #5 on the 2012 exam and #1 on the calculator portion of the 2013 exam. They are good examples, and follow the same processes outlined below.

1) Represent the system as a linear combination of matrices. Name those matrices y', A, and y.

2) Find matrices P and D, where $D = P^{-1}AP$ and D is diagonal. Make sure that the eigenvalue in $D_{1,1}$ matches the first column of P, and so on.

   Set up the following system:

   $$\begin{bmatrix} u_1' \\ \vdots \\ u_n' \end{bmatrix} = D \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$$ and multiply to find equations for u' in terms of u.

   Integrate both equations to form the matrix u.

   $$u' = nu \longrightarrow u = Ce^{nx}$$

3) Multiply Pu to find the general sol'n for y.

4) Use the given initial values to find a specific solution.

<u>USE THE EXAMPLES!!</u> They have more info.