Linear Algebra
Exam #1 (Chapters 1 & 2)
FALL 2016
Part 1 – GDC Allowed

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Date: 2016-09-13
Period: 7

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SHOW ALL WORK TO RECEIVE FULL CREDIT!!!

1. Solve the system using an inverse matrix:

$$x_1 + x_2 + x_3 + x_4 = 4$$
$$-x_1 + 2x_2 + x_3 = 0$$
$$2x_1 + 3x_2 + x_3 - x_4 = 6$$
$$-2x_1 + x_2 - 2x_3 + 2x_4 = -1$$

Note: You may use your GDC to find the inverse and perform any other necessary operations, but your work should include the inverse and show the steps used to solve the system.

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 2 & 1 & 0 \\ 2 & 3 & 1 & -1 \\ -2 & 1 & -2 & 2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 4 \\ 0 \\ 6 \\ -1 \end{bmatrix}}_{b}$$

$$\underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 1/5 & -2/5 & 1/5 & 0 \\ -2/35 & 4/35 & 8/35 & 1/7 \\ 11/35 & 13/35 & -9/35 & -2/7 \\ 19/35 & -3/35 & -6/35 & 1/7 \end{bmatrix}}_{A^{-1}} \underbrace{\begin{bmatrix} 4 \\ 0 \\ 6 \\ -1 \end{bmatrix}}_{b}$$

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix}}$$

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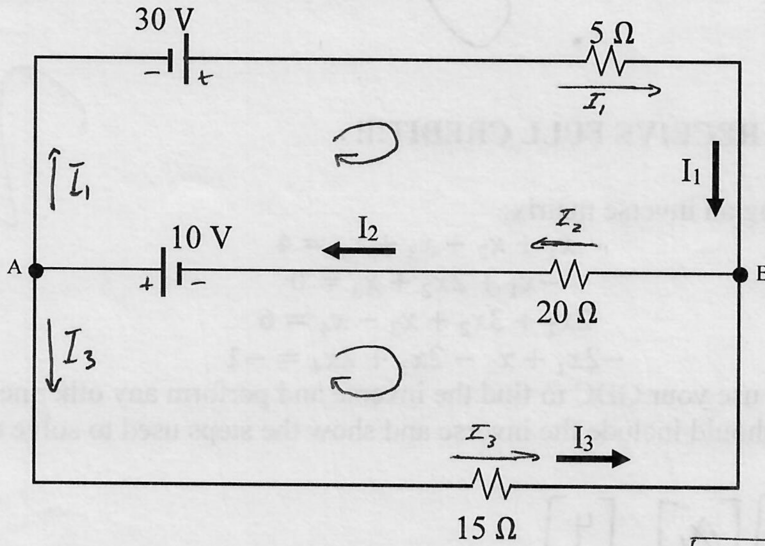
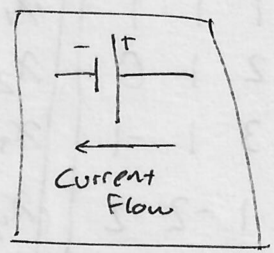
2. For the circuit shown below, write the corresponding system of equations and solve the system to find the currents flowing through the circuit.



$1: -48/19$

$2: -26/19$

$3: 22/19$



KCL

A: $I_2 = I_1 + I_3$
B: $I_1 + I_3 = I_2$
} same

KVL

Upper: $-40 = 5I_1 + 20I_2$

Lower: $+10 = -20I_2 - 15I_3$

$$
\begin{array}{c|c}
 & \begin{array}{ccc|c} I_1 & I_2 & I_3 & \end{array} \\
\begin{array}{c} \text{KCL:} \\ \text{Up:} \\ \text{Low:} \end{array} & \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 5 & 20 & 0 & -40 \\ 0 & -20 & -15 & +10 \end{array}\right]
\end{array}
\xrightarrow{\text{RREF}}
\left[\begin{array}{ccc|c} 1 & 0 & 0 & -48/19 \\ 0 & 1 & 0 & -26/19 \\ 0 & 0 & 1 & 22/19 \end{array}\right]
\begin{array}{c} I_1 \\ I_2 \\ I_3 \end{array}
$$

$I_1 = 48/19$ (opposite of direction drawn)

$I_2 = 26/19$ (" " " ")

$I_3 = 22/19$

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3. Find the cubic function that contains the points $(-2,-10), (-1,3), (1,5), \& (3,15)$.

$$p(x) = ax^3 + bx^2 + cx^1 + dx^0$$

$$\begin{array}{c|c} & \begin{array}{ccccc} x^0 & x^1 & x^2 & x^3 & p(x) \end{array} \\ \begin{array}{c} (-2,-10) \\ (-1,3) \\ (1,5) \\ (3,15) \end{array} & \left[\begin{array}{cccc|c} 1 & -2 & 4 & -8 & -10 \\ 1 & -1 & 1 & -1 & 3 \\ 1 & 1 & 1 & 1 & 5 \\ 1 & 3 & 9 & 27 & 15 \end{array}\right] \\ & \begin{array}{cccc} d & c & b & a \end{array} \end{array} \xrightarrow{\text{RREF}} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$d = 6$

$c = 0$

$b = -2$

$a = 1$

$$\boxed{p(x) = x^3 - 2x^2 + 6}$$

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4. A company produces Web design, software, and networking services. It requires \$0.40 of Web design, \$0.30 of software, and \$0.15 of networking to produce \$1 of Web design; it requires \$0.20 of Web design, \$0.35 of software, and \$0.10 of networking to produce \$1 of software; it requires \$0.45 of Web design, \$0.30 of software, and \$0.20 of networking to produce \$1 of networking. Suppose that the customers in the open sector have a demand for \$5400 worth of Web design, \$2700 worth of software, and \$900 worth of networking. What should the company produce to meet this need?

needed

prod

$$\begin{array}{cc} & \begin{array}{ccc} WD & S & NW \end{array} \\ \begin{array}{c} WD \\ S \\ NW \end{array} & \begin{bmatrix} 0.40 & 0.20 & 0.45 \\ 0.30 & 0.35 & 0.30 \\ 0.15 & 0.10 & 0.20 \end{bmatrix} \end{array} = C$$

Consumption

Leontief: $(I_3 - C)x = d$

$$(I_3 - C) = \begin{bmatrix} 0.60 & -.2 & -.45 \\ -.3 & 0.65 & -.3 \\ -.15 & -.1 & 0.80 \end{bmatrix}$$

$$x = (I_3 - C)^{-1} \begin{bmatrix} 5400 \\ 2700 \\ 900 \end{bmatrix}$$

demand

$$\boxed{x = \begin{bmatrix} \$19578.29 \\ \$16346.56 \\ \$6839.25 \end{bmatrix}}$$

← web
← software
← networking

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Exam #1 (Chapters 1 & 2) Name Date Period

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Part 2 – Scientific Calculator only

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USE THE METHODS SPECIFIED IN EACH PROBLEM, AND SHOW ALL STEPS TO RECEIVE FULL CREDIT!!

1. Given the following matrices A, B, C, D & E:

$$A = \begin{bmatrix} 0 & 3 & -5 \\ 1 & 2 & 6 \end{bmatrix}; \quad B = \begin{bmatrix} 4 & 1 & 0 \\ -2 & 3 & -2 \end{bmatrix}; \quad C = \begin{bmatrix} 4 & 1 \\ 6 & 2 \\ -2 & 3 \end{bmatrix}; \quad D = \begin{bmatrix} 2 & 4 \\ 5 & 3 \end{bmatrix}; \quad E = \begin{bmatrix} 3 & 6 & 2 \\ -9 & 0 & 4 \\ 3 & 3 & 1 \end{bmatrix}$$

a. Find DA:

$$\underset{(2\times 2)}{D} \; \underset{(2\times 3)}{A} = \begin{bmatrix} 4 & 14 & 14 \\ 3 & 21 & -7 \end{bmatrix}$$

b. Find CB:

$$\underset{(3\times 2)}{C} \; \underset{(2\times 3)}{B} = \begin{bmatrix} 14 & 7 & -2 \\ 20 & 12 & -4 \\ -14 & 7 & -6 \end{bmatrix}$$

c. Find 3A – 2B:

$$\begin{bmatrix} 0 & 9 & -15 \\ 3 & 6 & 18 \end{bmatrix} - \begin{bmatrix} 8 & 2 & 0 \\ -4 & 6 & -4 \end{bmatrix} = \begin{bmatrix} -8 & 7 & -15 \\ 7 & 0 & 22 \end{bmatrix}$$

d. Find D^{-1}:

$$\frac{1}{6-20}\begin{bmatrix} 3 & -4 \\ -5 & 2 \end{bmatrix} = \frac{-1}{14}\begin{bmatrix} 3 & -4 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} -3/14 & 2/7 \\ 5/14 & -1/7 \end{bmatrix}$$

e. Write a matrix **F** that would be added to E to transform E into I_3.

$$F = \begin{bmatrix} -2 & -6 & -2 \\ 9 & 1 & -4 \\ -3 & -3 & 0 \end{bmatrix}$$

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2. Write the system of equations as an augmented matrix, and solve the system by Gauss-Jordan elimination:

$$6x - y - z = 4$$
$$-12x + 2y + 2z = -8$$
$$5x + y - z = 3$$

Note: your work must show the matrix in the correct form prior to solving the system; please draw a box around this final matrix.

$$\left[\begin{array}{ccc|c} 6 & -1 & -1 & 4 \\ -12 & 2 & 2 & -8 \\ 5 & 1 & -1 & 3 \end{array}\right] \xrightarrow[R_2+2R_1]{R_1-R_3} \left[\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 5 & 1 & -1 & 3 \end{array}\right] \xrightarrow{R_3-5R_1} \left[\begin{array}{cccc} 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 11 & -1 & -2 \end{array}\right]$$

$$\xrightarrow[\frac{1}{11}R_2]{R_2 \leftrightarrow R_3} \boxed{\left[\begin{array}{cccc} 1 & -2 & 0 & 1 \\ 0 & 1 & -\frac{1}{11} & -\frac{2}{11} \\ 0 & 0 & 0 & 0 \end{array}\right]}$$

let $z=t$

$y - \frac{1}{11}t = -\frac{2}{11}$

$y = \frac{1}{11}t - \frac{2}{11}$

$x - 2\left(\frac{1}{11}t - \frac{2}{11}\right) = 1$

$x = 1 + \frac{2}{11}t - \frac{4}{11} = \cancel{}$

$x = \frac{7}{11} + \frac{2}{11}t$

$\boxed{\left(\frac{7}{11} + \frac{2}{11}t, \frac{1}{11}t - \frac{2}{11}, t\right)}$

Check: $6\left(\frac{7}{11} + \frac{2}{11}t\right) - \left(\frac{1}{11}t - \frac{2}{11}\right) - t = 4$

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3. Use Cramer's Rule to solve the system of equations:

$$2x + 3y = 6$$

$$x - y = \frac{1}{2}$$

$A = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$ $\det(A) = -2 - 3 = -5$

$A_1 = \begin{bmatrix} 6 & 3 \\ \frac{1}{2} & -1 \end{bmatrix}$ $\det(A_1) = -6 - \frac{3}{2} = \frac{-15}{2}$

$A_2 = \begin{bmatrix} 2 & 6 \\ 1 & \frac{1}{2} \end{bmatrix}$ $\det(A_2) = 1 - 6 = -5$

$$\boxed{x = \frac{-15/2}{-5} = \frac{3}{2} \quad y = \frac{-5}{-5} = 1}$$

Check ☑

4. Solve for x:

a. $\begin{vmatrix} 1 & 2 & x \\ -1 & 3 & 2 \\ 3 & -2 & 1 \end{vmatrix} = 0$

$\begin{matrix} 1 & 2 & 3 & 1 & 2 \\ -1 & 3 & 2 & -1 & 3 \\ 3 & -2 & 1 & 3 & -2 \end{matrix}$

Check ☑

$(3 + 12 + 2x) - (9x - 2 - 4) = 0$

$15 + 2x - 9x + 6 = 0$

$-7x = -21$

$\boxed{x = 3}$

$(3 + 12 + 6) - (27 - 4 - 2) = 0$

b. $\begin{vmatrix} x-1 & x \\ x+1 & 2 \end{vmatrix} = -8$

$3: \begin{vmatrix} 2 & 3 \\ 4 & 2 \end{vmatrix} = 4 - 12$ ✓

$2(x-1) - x(x+1) = -8$

$2x - 2 - x^2 - x = -8$

$x^2 - x - 6 = 0$

$(x-3)(x+2)$

$\boxed{x = 3, x = -2}$

$-2 \begin{vmatrix} -3 & -2 \\ -1 & 2 \end{vmatrix} = -6 - 2$ ✓

Check ☑

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5. Evaluate the following determinants using the designated method (your work should reflect that you used the indicated method):

a. $\begin{vmatrix} 4 & 0 & 0 & 0 \\ 6 & -5 & 0 & 0 \\ 1 & 3 & 1 & 0 \\ 1 & -2 & 7 & 3 \end{vmatrix}$; inspection $4(-5)(1)(3) = \boxed{-60}$

Ans: -336

b. $\begin{vmatrix} 1 & -1 & 8 & 4 \\ 2 & 6 & 0 & -4 \\ 2 & 0 & 2 & 6 \\ 0 & 2 & 8 & 0 \end{vmatrix}$; row-reduction $R_2 - 2R_1$, $R_3 - 3R_1$

$\begin{vmatrix} 1 & -1 & 8 & 4 \\ 0 & 8 & -16 & -12 \\ 0 & 3 & -22 & -6 \\ 0 & 2 & 8 & 0 \end{vmatrix}$ $R_2 - 3R_3 + R_2$

WTF

$\begin{vmatrix} 1 & -1 & 8 & 4 \\ 0 & 1 & 58 & 6 \\ 0 & 3 & -22 & -6 \\ 0 & 2 & 8 & 0 \end{vmatrix}$ WTF $R_3 - 3R_2$, $R_4 - 2R_2$

$\begin{vmatrix} 1 & -1 & 8 & 4 \\ 0 & 1 & 58 & 6 \\ 0 & 0 & -196 & -24 \\ 0 & 0 & -108 & 12 \end{vmatrix}$ $-\frac{1}{196}R_3$ (-196)

WTF

$\begin{vmatrix} 1 & -1 & 8 & 4 \\ 0 & 1 & 58 & 6 \\ 0 & 0 & 1 & 6/49 \\ 0 & 0 & -108 & 12 \end{vmatrix}$ $R_4 + 108R_3$

$\begin{vmatrix} 1 & -1 & 8 & 4 \\ 0 & 1 & 58 & 6 \\ 0 & 0 & 1 & 6/49 \\ 0 & 0 & 0 & \frac{1236}{49} \end{vmatrix} = \frac{1236}{49}$

$\left(\frac{1236}{49}\right)(-196) = \boxed{-4944}$

X -2

c. $\begin{vmatrix} 3 & 2 & 4 & -1 & 5 \\ -2 & 0 & 1 & 3 & 2 \\ 1 & 0 & 0 & 4 & 0 \\ 6 & 0 & 2 & -1 & 0 \\ 3 & 0 & 5 & 1 & 0 \end{vmatrix}$; cofactor expansion

col $j=2$: $2(-1)^{1+2} \begin{vmatrix} -2 & 1 & 3 & 2 \\ 1 & 0 & 4 & 0 \\ 6 & 2 & -1 & 0 \\ 3 & 5 & 1 & 0 \end{vmatrix} = -2(-206) = \boxed{412}$

using row $j=4$: $2(-1)^{1+4} \begin{vmatrix} 1 & 0 & 4 \\ 6 & 2 & -1 \\ 3 & 5 & 1 \end{vmatrix} = -2(103) = -206$

$(2+0+120)-(24+0-5) = 103$

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6. Let matrix A be a 5x5 matrix with a determinant of 4. Find each of the following:

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&5\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&0&5\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&\end{bmatrix} \qquad \begin{bmatrix}1&0\\0&3\end{bmatrix}\begin{bmatrix}1&0\\0&3\end{bmatrix}=\begin{bmatrix}1&0\\0&9\end{bmatrix}$$

$$\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&4\end{bmatrix} \qquad \begin{bmatrix}1&0\\ & \end{bmatrix}$$

a. $\det(A^2) = \boxed{16}$

b. $\det(3A) = 3^5 \cdot \det(A) = \boxed{972}$

c. $\det(A^{-1}) = \frac{1}{\det(A)} = \boxed{\frac{1}{4}}$

7. For the following system of equations:

$$x_1 + x_2 + x_3 = 4$$
$$x_3 = 2$$
$$(a^2 - 4)x_3 = a - 2$$

$$\left[\begin{array}{ccc|c}1&1&1&4\\0&0&1&2\\0&0&(a^2-4)&a-2\end{array}\right]$$

$a+2=1$
$a=-3$

$$\begin{bmatrix}1&1&1&4\\0&0&1&2\\0&0&1&\frac{1}{a+2}\end{bmatrix} \qquad 2=\frac{1}{a+2}$$

a. For what value(s) of *a* does the system have no solution?

~~$2(a^2-4)=a-2$~~
~~$2(a+2)(a-2)=(a-2)$~~
~~$2a+4=1$~~
~~$a=-\frac{3}{2}$~~

x_3 must $=2$.

$(a^2-4)(2)=a-2$

$(a+2)(2)=1$

$a=\frac{-3}{2}$

no sol. when this is untrue.

$\boxed{\text{for all } a \neq \frac{-3}{2}}$ -1 2 works

b. For what value(s) of *a* does the system have exactly one solution?

There are $\boxed{\text{no values}}$ of a for which the system has one solution because ~~[illegible]~~ x_1 or x_2 can be free; it is only necessary that $x_1+x_2=2$.

c. For what value(s) of *a* does the system have infinitely many solutions?

$\boxed{a=\frac{-3}{2}}$ -1 ~~$(t, 2-t, \frac{3}{2})$ for all t~~

$(t, 2-t, 2)$ for all t.

2

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8. For following matrix: $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$

a. Use the inversion algorithm to find the inverse.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 6 & -2 & -3 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3-6R_1]{R_2-R_1} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 4 & -3 & -6 & 0 & 1 \end{array}\right] \xrightarrow{R_3-4R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array}\right]$$

$$\xrightarrow{R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -3 & -3 & 1 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array}\right] \xrightarrow{R_1+R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -3 & 1 \\ 0 & 1 & 0 & -3 & -3 & 1 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array}\right]$$

inverse

answer

$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix} \begin{bmatrix} -2 & -3 & 1 \\ -3 & -3 & 1 \\ -2 & -4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

answer

b. Explain how you would check your answer.

$\boxed{AA^{-1} = I}$

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